OCF-networks with missing values

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Abstract. Similar to Bayesian networks, OCF-networks combine structural information encoded in a directed graph with qualitative information expressed by ranking degrees of (conditional) formulas. The benefits of such techniques are twofold: First, the high complexity of the semantical ranking functions approach is reduced substantially, and second, global ranks are obtained from local information. However, in many practical applications, even the local rankings are only available in parts, or not exactly in the format that is needed. In this paper, we apply inductive reasoning methods like System Z^{+} or c-representations, to fill up missing values in the local conditional tables. This allows the user to specify knowledge for such OCF-networks in its most appropriate and reliable form and leave the technical details to an inference engine.

1 Introduction

Uncertain and defeasible reasoning is often crucially based on appropriate semantical frameworks like, for example, probability theory that allow for a rich and meaningful representation of the problem domain under consideration while leaving enough semantical room for handling exceptions and nonmonotonic phenomena. One of these frameworks is provided by the theory of *ordinal conditional functions (OCF)* [1], also called *ranking functions*, that assign a degree of disbelief to any possible world. Ranking functions have become increasingly popular within the last decade, as they are essentially qualitative and more easily understandable than probabilities but share some nice features with probabilities. Most importantly, they provide proper interpretations for (meaningful, non-material) conditionals $(B|A)$ – *If A then plausibly B*, encoding a plausible relationship between their antecedents A and consequents B.

However, the drawback of most semantical approaches is their high complexity as query answering and inference procedures have to take (basically) all models into account. To make local computations considering only a subset of all variables possible, graphical structures like Bayesian networks [2] have proved extremely useful. Usually, they come along with a causal interpretation considering the parents of a variable as its (common) causes. For OCFs, a similar type of networks has been proposed in [3,4]. In these approaches, analogous to Bayesian networks, OCFs are factorized according to the structure of a graph, and local ranking tables involving only a few nodes serve to build a global ranking function. Still, as in Bayesian networks, the local ranking tables need full information of how plausible a literal is given all configurations of the parents of the respective variable. In application scenarios, often only partial information is available here, typically, the user only knows the plausibility of a variable given each cause separately and cannot say much about cases when some configuration of causes is present. To fill up information, often external combination rules like naive Bayes [5] are applied which, however, do not take the semantical structure of the problem under consideration into account.

In this paper, we propose methods to combine partial ranking information in an intensional way to come up with full local ranking tables so that the OCF-network and hence the induced global ranking function can be completely specified. The basic idea is to apply inductive conditional reasoning mechanisms like System Z+ [6] and c-representations [7] locally to find appropriate (complete) rankings for the respective subgraph, that is, a node and its parents, and extract from this semantical information the missing values in the local table of the child node. Similar approaches have been presented for Bayesian networks by making use of the maximum entropy principle [8,9,10]. Indeed, the maximum entropy distribution is a probabilistic c-representation for the given knowledge base [7], and for the OCF framework, inferences based on c-representations have also proved to satisfy all major properties of nonmonotonic reasoning [11]. Therefore we make use of high quality semantical methods to exploit the given partial information in an optimal way. This paper is an extended version of the paper [12]; in particular, we elaborated on formal properties of OCF-networks in much more detail. The rest of this paper is organized as follows: After short preliminaries in Section 2 we recall ranking functions in Section 3. Then, in Section 4, we introduce the systems to be used for inductive conditional reasoning, namely System Z^+ and c-representations. In Section 5 we elaborate on the concept of networks for ranking functions. We discuss why local ranks may not be available in the needed format for all vertices in the network and show how to solve this problem with the presented approaches in Section 6. Finally, we conclude in Section 6.

2 Preliminaries

Let $\Sigma = \{V_1, \ldots, V_n\}$ be a set of propositional atoms and a *literal* a positive or negative atom representing variables in their positive resp. negated form; for a specific, nevertheless undetermined, outcome of V_i, we write $\dot{v}_i \in \{v_i, \overline{v}_i\}$.

The set of formulas $\mathfrak{L}$ over Σ joined with the symbols for tautology ($\top$) and contradiction ($\bot$), with the connectives $\wedge$ (*and*), $\vee$ (*or*) and $\neg$ (*not*) shall be defined in the usual way. For $A, B \in \mathfrak{L}$, we usually omit the connective $\wedge$ and write AB instead of $A \wedge B$ as well as indicate negation by overlining, that is, $\overline{A}$ means $\neg A$.

Interpretations, or *possible worlds*, are also defined in the usual way; the set of all possible worlds is denoted by Ω. We often use the 1-1 association between worlds and *complete conjunctions*, that is, conjunctions of literals where every variable $V_i \in \Sigma$ appears exactly once. A model ω of a propositional formula $A \in \mathfrak{L}$ is a possible world that satisfies A, written as $\omega \models A$. The set of all models $\omega \models A$ is denoted by $Mod(A)$. For formulas $A, B \in \mathfrak{L}$, A *entails* B, written as $A \models B$, iff $Mod(A) \subseteq Mod(B)$, that is, if and only if for all $\omega \in \Omega$, $\omega \models A$ implies $\omega \models B$. For sets of formulas $\mathcal{A} \subseteq \mathfrak{L}$ we have $Mod(\mathcal{A}) = \bigcap_{A \in \mathcal{A}} Mod(A)$.

$\kappa(\omega) = 4$	$p\overline{b}f$
$\kappa(\omega) = 2$	pbf $\quad p\overline{bf}$
$\kappa(\omega) = 1$	$pb\overline{f}$ $\quad \overline{p}b\overline{f}$
$\kappa(\omega) = 0$	$\overline{p}bf$ $\quad \overline{p}\overline{b}f$ $\quad \overline{p}\overline{bf}$

ω	pbf	$pb\overline{f}$	$p\overline{b}f$	$p\overline{b}\,\overline{f}$	$\overline{p}bf$	$\overline{p}b\overline{f}$	$\overline{p}\,\overline{b}f$	$\overline{p}\,\overline{b}\,\overline{f}$
$\kappa(\omega)$	2	1	4	2	0	1	0	0
$\kappa_Z(\omega)$	4	1	12	12	0	1	0	0
$\kappa^c_{\mathcal{R}}(\omega)$	3	1	14	11	0	1	0	0

Fig. 1. Ranking model κ of the penguin example $\mathcal{R} = \{(f|b), (\overline{f}|p), (b|p)\}$ given as worlds stacked by their plausibility (left) and in tabular form (right), and ranking models κ_Z and $\kappa^c_{\mathcal{R}}$ of the annotated penguin knowledge base in Example 1.

A *conditional* $(B|A)$ with $A, B \in \mathfrak{L}$ encodes a defeasible rule "if A then *usually* B" with the trivalent evaluation $[\![(B|A)]\!]_\omega = true$ if and only if $\omega \models AB$ (verification), $[\![(B|A)]\!]_\omega = false$ if and only if $\omega \models A\overline{B}$ (falsification) and (non-applicability) $[\![(B|A)]\!]_\omega = undefined$ if and only if $\omega \models \overline{A}$ [13,11]. The language of all conditionals over $\mathfrak{L}$ is denoted by $(\mathfrak{L} \mid \mathfrak{L})$.

We denote by $\mathcal{R}$ a finite set of conditionals $\mathcal{R} = \{(B_1|A_1), \ldots, (B_n|A_n)\} \subseteq (\mathfrak{L} \mid \mathfrak{L})$. A conditional $(B|A)$ is *tolerated* by $\mathcal{R}$ if and only if there is a world $\omega \in \Omega$ such that $\omega \models AB$ and $\omega \models A_i \Rightarrow B_i$ for every $1 \leq i \leq n$. $\mathcal{R}$ is *consistent* if and only if for every nonempty subset $\mathcal{R}' \subseteq \mathcal{R}$ there is a conditional $(B|A) \in \mathcal{R}'$ that is tolerated by $\mathcal{R}'$ [3]. We call such a consistent $\mathcal{R}$ a *knowledge base* and it shall represent the knowledge an agent uses as a base for reasoning.

3 Ranking Functions (OCF)

An *ordinal conditional function* (OCF, [1]), also called *ranking function*, is a function $\kappa : \Omega \to \mathbb{N}_0^\infty$ with $\kappa^{-1}(0) \neq \varnothing$ which maps each world $\omega \in \Omega$ to a degree of implausibility $\kappa(\omega)$, that is, if for two possible worlds $\omega, \omega' \in \Omega$ it holds that $\kappa(\omega) < \kappa(\omega')$ then ω' is believed to be less plausible than ω. Ranks of formulas $A \in \mathfrak{L}$ are calculated as $\kappa(A) = \min\{\kappa(\omega) \mid \omega \models A\}$. For conditionals $(B|A)$ a rank is defined via $\kappa(B|A) = \kappa(AB) - \kappa(A)$. A ranking function κ is a *(ranking) model* of a conditional $(B|A)$, written $\kappa \models (B|A)$ if and only if $\kappa(AB) < \kappa(A\overline{B})$, that is, if and only if AB is more plausible than $A\overline{B}$. Figure 1 is an example for a ranking model of the knowledge base for the well-known *penguin-example*, $\mathcal{R} = \{(f|b), (\overline{f}|p), (b|p)\}$, encoding the rules "*b*irds usually $f$ly", "*p*enguins usually do not $f$ly" and "*p*enguins usually are *b*irds". If $\kappa \models (B|A)$ we also say that $(B|A)$ is *believed in* κ.

Sometimes it ie convenient to express how strongly a conditional is believed.

Definition 1 (Firmness [1]). *A proposition A is believed in an OCF κ with* firmness *m, $m \in \mathbb{N}, m \geq 1$, in symbols $\kappa \models A[m]$, if and only if $\kappa(\overline{A}) \geq m$. A conditional $(B|A)$ is* believed with firmness *m ($\kappa \models (B|A)[m]$), if and only if $\kappa(\overline{B}|A) \geq m$.*

Equivalently, $(B|A)$ is believed in κ with firmness m iff $\kappa(AB) + m \leq \kappa(A\overline{B})$. Moreover, κ is a model of $(B|A)$ if the conditional is believed with firmness $m = 1$, that is, if $\kappa(AB) + 1 \leq \kappa(A\overline{B})$ which is equivalent to $\kappa(AB) < \kappa(A\overline{B})$. We illustrate the notion of firmness with the following example.

Example 1. Let $\mathcal{R} = \{(f|b)[1], (\overline{f}|p)[2], (b|p)[10]\}$ be the penguin knowledge base with annotated conditionals. Two OCFs, κ_Z and $\kappa^c_{\mathcal{R}}$, that are models of $\mathcal{R}$ are shown in Fig. 1.

Note that $\kappa \models A[m]$ if and only if $\kappa \models (A|\top)[m]$ since $\kappa(\top) = 0$, so (plausible) formulas can be considered as a special case of conditionals. Hence, we focus on conditional knowledge bases in this paper, keeping in mind that such knowledge bases may also contain plausible propositions. Moreover, we presuppose $m \geq 1$ in this paper since $\kappa \models (B|A)[m]$ should imply in particular $\kappa \models (B|A)$. Nevertheless, the case $m = 0$ is interesting but requires further considerations as we might have $\kappa(AB) = \kappa(A\overline{B})$, or $\kappa \models (\overline{B}|A)$. To keep the technical details as clear and simple as possible, we leave the case $m = 0$ for future work.

Definition 2 (Admissibility). *A ranking function is* admissible regarding a knowledge base *of conditionals* $\mathcal{R} = \{(B_1|A_1)[m_1], \ldots, (B_n|A_n)[m_n]\}$ *annotated with firmness values (formally,* $\kappa \models \mathcal{R}$*) if and only if mbox*$\kappa \models (B_i|A_i)[m_i]$ *for every* $1 \leq i \leq n$*.*

For the networks to be considered in this paper, we need a notion of *conditional independence* regarding ranking functions [1], as it is necessary for Bayes networks.

Definition 3 (Conditional κ-independence [1]). *Let* $\mathbf{A}$,$\mathbf{B}$,$\mathbf{C}$ *be disjoint sets of variables.* $\mathbf{A}$ *is* (conditionally) κ-independent of $\mathbf{B}$ given $\mathbf{C}$ *, written* $\mathbf{A} \perp\!\!\!\perp_\kappa \mathbf{B} \mid \mathbf{C}$ *if and only if* $\kappa(\mathbf{ab}|\mathbf{c}) = \kappa(\mathbf{a}|\mathbf{c}) + \kappa(\mathbf{b}|\mathbf{c})$ *for all complete conjunctions* $\mathbf{a}$,$\mathbf{b}$,$\mathbf{c}$ *built over* $\mathbf{A}$,$\mathbf{B}$,$\mathbf{C}$*, respectively.*

κ-independence can be characterized equivalently as in the probabilistic case:

Lemma 1. *For all disjoint sets of variables* $\mathbf{A}$,$\mathbf{B}$,$\mathbf{C}$*,* $\mathbf{A}$ *is* κ*-independent of* $\mathbf{B}$ *given* $\mathbf{C}$ *(*$\mathbf{A} \perp\!\!\!\perp_\kappa \mathbf{B} \mid \mathbf{C}$*) if and only if* $\kappa(\mathbf{a}|\mathbf{bc}) = \kappa(\mathbf{a}|\mathbf{c})$ *for all complete conjunctions* $\mathbf{a}$,$\mathbf{b}$,$\mathbf{c}$ *over* $\mathbf{A}$,$\mathbf{B}$,$\mathbf{C}$*, respectively.*

Proof. $\mathbf{A} \perp\!\!\!\perp_\kappa \mathbf{B} \mid \mathbf{C}$ means $\kappa(\mathbf{ab}|\mathbf{c}) = \kappa(\mathbf{a}|\mathbf{c}) + \kappa(\mathbf{b}|\mathbf{c})$, therefore $\kappa(\mathbf{abc}) - \kappa(\mathbf{c}) = \kappa(\mathbf{a}|\mathbf{c}) + \kappa(\mathbf{bc}) - \kappa(\mathbf{c})$. This is equivalent to $\kappa(\mathbf{abc}) - \kappa(\mathbf{bc}) = \kappa(\mathbf{a}|\mathbf{c})$, hence $\kappa(\mathbf{a}|\mathbf{bc}) = \kappa(\mathbf{a}|\mathbf{c})$, which was to be shown.

4 Inductive conditional reasoning

Let $\mathcal{R} = \{(B_1|A_1)[m_1], \ldots, (B_n|A_n)[m_n]\}$ be an annotated knowledge base. Taking all admissible ranking functions into account yields quite a weak inference from $\mathcal{R}$, because in this case, the set of inferences that could be drawn from $\mathcal{R}$ is the intersection of all $\mathcal{R}$-admissible OCFs, that is, the sceptical inference relation regarding $\mathcal{R}$. A popular approach to obtain more informative inferences from $\mathcal{R}$ is realised by selecting a "best" ranking model of $\mathcal{R}$ to be used for further inferences. In the following, we recall two approaches to obtain such a "best" ranking function for inductive model-based inference.

4.1 System $\mathbf{Z}^{+}$

A well known approach to compute a ranking function given an annotated knowledge base $\mathcal{R} = \{(B_1|A_1)[m_1], \dots, (B_n|A_n)[m_n]\}$ is System Z^{+} [6] which is a generalization of System Z [14]. Here, the tolerance condition (cf. Section 2) is extended to firmness annotated conditionals such that $\mathcal{R}$ tolerates $(D|C)[o]$ if the knowledge base $\mathcal{R}^* = \{(B|A)|(B|A)[m] \in \mathcal{R}\}$ tolerates $(D|C)$. We start by selecting the set of conditionals $\mathcal{R}_0 \subseteq \mathcal{R}$ which are tolerated by the whole knowledge base, so $\mathcal{R}_0$ consists of all conditionals $(B|A)[m]$ with the property that there is a world ω such that $\omega \models AB$ and $\omega \models (A_i \Rightarrow B_i)$ for each $(B_i|A_i)[m] \in \mathcal{R}$. These conditionals get a Z-value identical to their firmness, that is, $Z(B_i|A_i) = m_i$ for all $(B_i|A_i) \in \mathcal{R}_0$. We initialize $\mathcal{RZ}$ to $\mathcal{RZ} = \mathcal{R}_0$. In the iteration step we select the set of worlds $\Omega_{\mathcal{RZ}}$ which solely falsify conditionals in $\mathcal{RZ}$ and verify at least one conditional not in $\mathcal{RZ}$. Each of these worlds is assigned a temporal κ^*_Z-rank calculated as $\kappa^*_Z(\omega) = \max\limits_{(B_i|A_i)\in\mathcal{RZ}} \{Z(B_i|A_i)|\omega \models A_i\overline{B}_i\} + 1$.

From $\Omega_{\mathcal{RZ}}$ we take a world ω^* with the smallest κ^*_Z-value, that is, a world $\omega^* \in \Omega_{\mathcal{RZ}}$ such that $\kappa^*_Z(\omega^*) = \min_{\omega\in\Omega_{\mathcal{RZ}}}\{\kappa^*_Z(\omega)\}$. By construction, for each $\omega \in \Omega_{\mathcal{RZ}}$ there is at least one conditional $(B_i|A_i)[m_i] \in \mathcal{R} \setminus \mathcal{RZ}$ that is verified by ω. To each of these conditionals we assign the Z-value $Z(B_i|A_i) = \kappa^*_z(\omega^*) + m_i$ and add $(B_i|A_i)$ to $\mathcal{RZ}$, obtaining a new set $\mathcal{RZ}$ with which we start the iteration again until $\mathcal{RZ} = \mathcal{R}$. For further details and theoretical background, confer [6].

There is a world ω with $\omega \models AB$ for every conditional $(B|A)[m] \in \mathcal{R}$, if the starting knowledge base $\mathcal{R}$ is consistent. This world either does not falsify any conditional $(B_i|A_i)[m] \in \mathcal{R}$, then $(B|A)$ is an element of $\mathcal{R}_0$, or there is a conditional $(D|C)[n] \in \mathcal{R}$ with $\omega \models C\overline{D}$, but then, ω is chosen as a world in $\Omega_{\mathcal{RZ}}$ at a time after $(D|C)$ was added to $\mathcal{RZ}$. By this we get an associated Z-value $Z(B_i|A_i)$ for all the conditionals in $\mathcal{R}$ and from these values we obtain a ranking function κ_Z defined as

$$\kappa_Z(\omega) = \begin{cases} 0 & \text{iff } \omega \text{ does not falsify any } (B_i|A_i) \\ \max\limits_{\omega\models(A_i\overline{B}_i)} \{Z(B_i|A_i)\} & \text{otherwise.} \end{cases} \quad (1)$$

Note that, differently from the original approach [6], instead of setting a world's rank to $\kappa_Z(\omega) = \max_{\omega\models(A_i\overline{B}_i)}\{Z(B_i|A_i)\} + 1$, we set this rank to the value of $\kappa_Z(\omega) = \max_{\omega\models(A_i\overline{B}_i)}\{Z(B_i|A_i)\}$, as for the admissibility of κ wrt. $(B|A)$ ($\kappa \models (B|A)[m]$), we require $\kappa(AB) + m \leq \kappa(A\overline{B})$ and not, like in [6], $\kappa(AB) + m < \kappa(A\overline{B})$, but presuppose $m > 0$.

Example 2 (System Z^{+} penguins). We use the knowledge base from Example 1 to illustrate how this framework works, so let $\mathcal{R} = \{(f|b)[1], (\overline{f}|p)[2], (b|p)[10]\}$. For the first step, we get $\mathcal{R}_0 = \{(f|b)\} = \mathcal{RZ}$, so $Z(f|b) = 1$. We obtain $\Omega_{\mathcal{RZ}} = pb\overline{f}$ and $\kappa^*_Z = 2$. We have $pb\overline{f} \models p\overline{f}$ as well as $pb\overline{f} \models bp$ and we get $Z(\overline{f}|p) = 2 + 2 = 4$ and $Z(b|p) = 2 + 10 = 12$. The resulting ranking function κ_Z is shown in Figure 1.

4.2 c-representations

The framework of c-representation [11] generates ranking functions $\kappa^c_{\mathcal{R}}$ for knowledge bases $\mathcal{R}$ that are $\mathcal{R}$-admissible and are based on the conditionals in the knowledge base and their structure, solely. In this section, we rejudge this approach.

Definition 4 (c-representation [11]). *A c-representation of a knowledge base defined as* $\mathcal{R} = \{(B_1|A_1)[m_1], \ldots, (B_n|A_n)[m_n]\}$ *is an OCF of the form*

$$\kappa^c_{\mathcal{R}}(\omega) = \sum_{\substack{i=1 \\ \omega \models A_i\overline{B}_i}}^{n} \kappa_i^-, \qquad \kappa_i^- \in \mathbb{N}_0$$

where the values κ_i^- *are penalty points for falsifying conditionals and have to be chosen to make* $\kappa^c_{\mathcal{R}}$ $\mathcal{R}$*-admissible, that is for all* $1 \leq i \leq n$ *it holds that* $\kappa^c_{\mathcal{R}} \models (B_i|A_i)[m_i]$*. This is the case if and only if [7], (cf. Definition 1):*

$$\kappa_i^- \geq m_i + \min_{\omega \models A_i B_i} \left\{ \sum_{\substack{i \neq j \\ \omega \models A_j \overline{B}_j}} \kappa_j^- \right\} - \min_{\omega \models A_i \overline{B}_i} \left\{ \sum_{\substack{i \neq j \\ \omega \models A_j \overline{B}_j}} \kappa_j^- \right\} \qquad (2)$$

A minimal *c-representation is obtained by choosing* κ_i^- *minimally for all* $1 \leq i \leq n$*. (Note that there may be several different minimal c-representations for a given* $\mathcal{R}$*.)*

Example 3 (c-represented penguins). We use the knowledge base from Example 1 to illustrate how this framework works, so let $\mathcal{R} = \{(f|b)[1], (\overline{f}|p)[2], (b|p)[10]\}$. For the κ_i^- values of a c-representation we get, according to inequation (2), $\kappa_1^- \geq 1 + \min\{\kappa_2^-, 0\} - \min\{0\} = 1$, $\kappa_2^- \geq 2 + \min\{\kappa_1^-, \kappa_3^-\} - \min\{0\} = 2 + \min\{\kappa_1^-, \kappa_3^-\}$ and $\kappa_3^- \geq 10 + \min\{\kappa_1^-, \kappa_2^-\} - \min\{0\} = 10 + \min\{\kappa_1^-, \kappa_2^-\}$. This leads to a minimal c-representation with $\kappa_1^- = 1$, $\kappa_2^- = 3$, $\kappa_3^- = 11$ and the OCF $\kappa^c_{\mathcal{R}}$ shown in Fig. 1.

5 OCF-Networks

In this section, we elaborate on the concept of networks for OCFs. First approaches that make crucial use of the idea of causality have been presented in [3,4]. However, like in Bayesian networks, causal interpretations are not mandatory for such networks although they support appropriate modellings of the problem domain. More importantly, it is the idea of conditional independence that provides the basis for factorising OCFs, i.e., for local representations of global ranking functions. So, we prefer to develop the approach of OCF-networks in full analogy to Bayesian networks (as far as possible), making assumptions underlying the works [3,4] explicit.

Let $\Gamma = \langle \mathcal{V}, \mathcal{E} \rangle$ be a directed, acyclic graph (DAG) with a set of vertices $\mathcal{V} = \{V_1, \ldots, V_n\}$ and a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. We define the *parents* of a vertex V, $pa(V)$, as the direct predecessors of V (i.e., $pa(V) = \{V' | (V', V) \in \mathcal{E}\}$) and the *descendants* of V, $desc(V)$, as the set of vertices V' for which a path from V to V' exists in $\mathcal{E}$. The set of *non-descendants* of V is the set of all vertices that are neither the parents nor the descendants of V, nor V itself, so $nd(V) = \mathcal{V} \setminus (desc(V) \cup \{V\} \cup pa(V))$.

To connect a DAG with ranking information we define an OCF-Network as follows:

Definition 5 (OCF-Network). *A DAG $\Gamma = \langle \Sigma, \mathcal{E} \rangle$ over a set of propositional atoms Σ is an* OCF-network *if each vertex $V \in \Sigma$ is annotated with a table of local rankings $\kappa_V(V|pa(V))$ with (local) ranking values specified for every configuration of V and $pa(V)$. According to the definition of ranking functions the local rankings must be normalised, i.e.,*

$$\min_{\dot{v}} \{\kappa(\dot{v}|pa(V))\} = 0 \qquad \text{for every configuration of } pa(V), \tag{3}$$

and for the local rankings to provide a global picture, we also need

$$\min_{\omega} \sum \kappa_V(V(\omega)|pa(V)(\omega)) = 0, \tag{4}$$

to be fulfilled where $V(\omega)$ resp. $pa(V)(\omega)$ indicates the outcome $v \in dom(V)$ with $\omega \models v$ resp. the configuration p of the variables in $pa(V)$ with $\omega \models p$.

The local ranking information in Γ can be used to define a global ranking function κ over Σ by applying the idea of stratification [3]: A ranking function κ is *stratified* relative to an OCF-network Γ if and only if

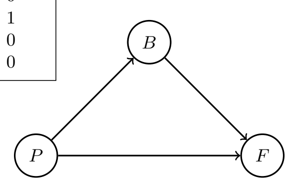
$$\kappa(\omega) = \sum \kappa_V(V(\omega)|pa(V)(\omega)), \tag{5}$$

for every world ω. With this stratifiction, given the tables of local rankings, we can generate a stratified OCF by formula (5). Condition (4) ensures that κ is indeed an OCF.

Example 4. As an illustration we use the penguin example already presented in Example 3 with a graph set up according to [3] and local conditional ranking values calculated as conditional ranks from the ranking function given in Example 3 shown in Figure 2, i.e., $\kappa_P(B|P) = \kappa(B|P)$, $\kappa_F(F|PB) = \kappa_(F|PB)$, $\kappa_P(P) = \kappa(P)$.

$B\|P$	$\kappa_P(B\|P)$
$b\|p$	0
$\overline{b}\|p$	1
$b\|\overline{p}$	0
$\overline{b}\|\overline{p}$	0

P	$\kappa_P(P)$
p	1
$\overline{p}$	0



$F\|PB$	$\kappa_F(F\|PB)$
$f\|b\,p$	1
$\overline{f}\|b\,p$	0
$f\|b\,\overline{p}$	0
$\overline{f}\|b\,\overline{p}$	0
$f\|\overline{b}\,p$	2
$\overline{f}\|\overline{b}\,p$	0
$f\|\overline{b}\,\overline{p}$	0
$\overline{f}\|\overline{b}\,\overline{p}$	0

Fig. 2. Network of the penguin-example.

Conversely, given a DAG Γ with vertices Σ and an OCF κ over Σ such that each vertex $V \in \Sigma$ is κ-independent of its non-descendants given its parents, we obtain a stratification of κ relative to Γ. This is stated in the following proposition.

Proposition 1. *Let Σ be a propositional alphabet and $\Gamma = \langle \Sigma, \mathcal{E} \rangle$ be a DAG. Let $\Sigma = \{V_1, \ldots, V_n\}$ be enumerated such that for each $V_i \in \Sigma$ we have $pa(V_i) \subseteq \{V_1, \ldots, V_{i-1}\}$. Let κ be an OCF over Σ such that $V \perp\!\!\!\perp_\kappa nd(V) \mid pa(V)$ for all $V \in \Sigma$. Then it holds that*

$$\kappa(V_1, \ldots, V_n) = \sum_{i=1}^{n} \kappa(V_i | pa(V_i)). \tag{6}$$

Proof. Let Σ, Γ, κ be as presupposed in the lemma above. Then

$$\begin{aligned}
\kappa(V_1, \ldots, V_n) =& \kappa(V_1, \ldots, V_n) - \kappa(V_1, \ldots, V_{n-1}) + \kappa(V_1, \ldots, V_{n-1}) \\
& - \kappa(V_1, \ldots, V_{n-2}) + \kappa(V_1, \ldots, V_{n-2}) - \ldots - \kappa(V_1) + \kappa(V_1) \\
=& \kappa(V_n | V_1, \ldots, V_{n-1}) + \kappa(V_{n-1} | V_1, \ldots, V_{n-2}) + \ldots + \kappa(V_1) \\
=& \kappa(V_1) + \sum_{i=2}^{n} \kappa(V_i | V_1, \ldots, V_{i-1}).
\end{aligned}$$

By presupposition, $pa(V_i) \subseteq \{V_1, \ldots, V_{i-1}\}$ and $V_i \perp\!\!\!\perp_\kappa nd(V_i) \mid pa(V_i)$ for all V_i. With lemma 1 we obtain the equality $\kappa(V_i | V_1, \ldots, V_{i-1}) = \kappa(V_i | pa(V_i))$. Therefore for the joint ranking function we have

$$\kappa(V_1, \ldots, V_n) = \sum_{i=1}^{n} \kappa(V_i | pa(V_i)) \qquad \text{for every} \qquad 1 \leq i \leq n$$

which was to be shown.

Hence, a ranking function that implements the conditional independence assumptions of a network Γ can be stratified relative to Γ. Whether a stratified ranking function is admissible with respect to the ranking tables is settled by the next theorem.

Theorem 1. *Let W be a variable in Σ with a fixed value $\dot{w}$ of W, let $\dot{p}_w$ be a fixed configuration of the variables in $pa(W)$. For a ranking function κ stratified according to equation (5) the conditional ranking values $\kappa(\dot{w} | \dot{p}_w)$ are identical to the local ranking values $\kappa_W(\dot{w} | \dot{p}_w)$.*

Proof. According to the definitions of OCF and conditional ranking values in Section 3 we have

$$\begin{aligned}
\kappa(\dot{w} | \dot{p}_w) &= \kappa(\dot{w}\dot{p}_w) - \kappa(\dot{p}_w) \\
&= \min_{\omega \models \dot{w}\dot{p}_w} \{\kappa(\omega)\} - \min_{\omega \models \dot{p}_w} \{\kappa(\omega)\}.
\end{aligned}$$

With the stratification of equation (5), this rewrites to

$$\kappa(\dot{w} | \dot{p}_w) = \min_{\omega \models \dot{w}\dot{p}_w} \left\{ \sum_{V} \kappa_V \left(V(\omega) | pa(V)(\omega) \right) \right\} - \min_{\omega \models \dot{p}_w} \left\{ \sum_{V} \kappa_V \left(V(\omega) | pa(V)(\omega) \right) \right\}.$$

The value of $\kappa_W(\dot{w} | \dot{p}_w)$ is fixed in every sum of the first $\min$term and hence can be extracted. For the first $\min$terms we obtain with the normalisation condition (3) that for every vertex C in the set of children of W there is a configuration $\dot{c}$ of C such that $\kappa_C(\dot{c}|\dot{w}) = 0$, which holds iteratively for the children of C, so in the above formula, we have $\min\limits_{\omega \models \dot{w}\dot{p}_w} \left\{ \sum\limits_{V \in desc(W)} \kappa_V(V(\omega)|pa(V)(\omega)) \right\} = 0$. Since the configuration of each vertex apart from W and W's parents is not fixed and the configuration of each variable is independent from the others, the actual *minimum* is achieved when a configuration as sketched above is chosen, hence the descendants of W can be ignored for the first $\min$term. So we have

$$\kappa(\dot{w}|\dot{p}_w) = \min_{\omega \models \dot{w}\dot{p}_w} \left\{ \begin{array}{l} \kappa_W(\dot{w}|\dot{p}_w) \\ + \sum\limits_{V \in (nd(W) \cup pa(W))} \kappa_V\left(V(\omega)|pa(V)(\omega)\right) \\ + \sum\limits_{V \in desc(W)} \kappa_V\left(V(\omega)|pa(V)(\omega)\right) \end{array} \right\}$$

$$- \min_{\omega \models \dot{p}_w} \left\{ \sum_V \kappa_V\left(V(\omega)|pa(V)(\omega)\right) \right\}$$

$$= \kappa_W(\dot{w}|\dot{p}_w) + \min_{\omega \models \dot{w}\dot{p}_w} \left\{ \sum_{V \setminus (\{W\} \cup desc(W))} \kappa_V\left(V(\omega)|pa(V)(\omega)\right) \right\}$$

$$- \min_{\omega \models \dot{p}_w} \left\{ \sum_V \kappa_V\left(V(\omega)|pa(V)(\omega)\right) \right\}.$$

The value of vertices $V \in (nd(W) \cup pa(W))$ can be chosen indepently of the value W, while $\dot{p}_w$ is fixed, hence the minimum of the sum over this values ist constant for the term. It therefore can be extracted and will be called $Const$ in the following, so the equation can be written as

$$\kappa(\dot{w}|\dot{p}_w) = \kappa_W(\dot{w}|\dot{p}_w) + Const - \min_{\omega \models \dot{p}_w} \left\{ \sum_V \kappa_V\left(V(\omega)|pa(V)(\omega)\right) \right\}.$$

We now look at the second $\min$term. Here, W is not fixed so this variable can be chosen freely. Naturally, the minimum of this term is either the minimal sum with W fixed to w or to $\overline{w}$, so we can rewrite this to

$$\min_{\omega \models \dot{p}_w} \left\{ \sum_V \kappa_V\left(V(\omega)|pa(V)(\omega)\right) \right\}$$

$$= \min \left\{ \begin{array}{l} \min\limits_{\omega \models w\dot{p}_w} \left\{ \sum\limits_V \kappa_V\left(V(\omega)|pa(V)(\omega)\right) \right\}^{\Sigma_1}, \\ \min\limits_{\omega \models \overline{w}\dot{p}_w} \left\{ \sum\limits_V \kappa_V\left(V(\omega)|pa(V)(\omega)\right) \right\}^{\Sigma_2} \end{array} \right\}.$$

We continue our elaborations on the case Σ_1, first. We partition the summation condition V into in the set of W's descendants, W itself and all other vertices $(nd(W) \cup$

$pa(W)$) and obtain for Σ_1 the term

$$\Sigma_1 = \min_{\omega \models w\dot{p}_w} \left\{ \begin{array}{l} \sum_{V \in (nd(W) \cup pa(W))} \kappa_V(V(\omega)|pa(V)(\omega)) + \kappa_W(w|\dot{p}_w) \\ + \sum_{V \in desc(W)} \kappa_V(V(\omega)|pa(V)(\omega)) + \end{array} \right\}.$$

The minimum can be split, since the values of nodes in $(nd(W) \cup pa(W))$ can be chosen independently from those in $desc(W)$. $\kappa_W(w|\dot{p}_w)$ can be extracted since the values of W and $\dot{p}_w$ are fixed in the min. The sum over $V \in (nd(W) \cup pa(W))$ is independent of the interpretation of W, this minimum is the same constant $Const$ as above. The minimum of the sum over $V \in desc(W)$ is 0 as discussed above, so for this minimum we get $\Sigma_1 = Const + \kappa_W(w|\dot{p}_w)$. We extend this deliberations to Σ_2 and obtain similary $\Sigma_2 = Const + \kappa_W(\overline{w}|\dot{p}_w)$. So finally we optain the following for the second minterm.

$$\begin{aligned} &= \min \{Const + \kappa_W(w|\dot{p}_w), Const + \kappa_W(\overline{w}|\dot{p}_w)\} \\ &= Const + \min \{\kappa_W(w|\dot{p}_w), \kappa_W(\overline{w}|\dot{p}_w)\} \end{aligned}$$

We have $\min \{\kappa_W(w|\dot{p}_w), \kappa_W(\overline{w}|\dot{p}_w)\} = 0$ by the normalisation condition, therefore the overall equation is rewritten to

$$\kappa(\dot{w}|\dot{p}_w) = \kappa_W(\dot{w}|\dot{p}_w) + Const - Const.$$

And we obtain $\kappa(\dot{w}|\dot{p}_w) = \kappa_W(\dot{w}|\dot{p}_w)$ as proposed. □

Theorem 2. *Let $\Gamma = \langle \Sigma, \mathcal{E} \rangle$ be an OCF-network with local ranking tables $\kappa_V(V|pa(V))$ for every vertex $V \in \Sigma$ and stratified OCF κ according to (5). Then the local directed Markov property holds, that is, $V \perp\!\!\!\perp_\kappa nd(V) \mid pa(V)$ for each node V.*

Proof. Let Γ, κ_V and κ be as described in the theorem. For each node V, let $\dot{n}_v$ be a configuration of the variables $nd(V)$ and $\dot{p}_v$ be a configuration of the variables $pa(V)$. We consider a fixed but arbitrary variable W in Σ. We will show that

$$\kappa(\dot{w}\dot{n}_w|\dot{p}_w) = \kappa(\dot{w}|\dot{p}_w) + \kappa(\dot{n}_w|\dot{p}_w)$$

for every configuration of $\dot{w}$, $\dot{n}_w$ and $\dot{p}_w$, this establishes the local Markov property as claimed. From theorem 1 we obtain $\kappa(\dot{w}|\dot{p}_w) = \kappa_W(\dot{w}|\dot{p}_w)$, so by the definition of conditional ranks, it is equivalent to show $\kappa(\dot{w}\dot{n}_w\dot{p}_w) = \kappa_W(\dot{w}|\dot{p}_w) + \kappa(\dot{n}_w\dot{p}_w)$. We consider the left hand side first:

$$\begin{aligned} \kappa(\dot{w}\dot{n}_w\dot{p}_w) &= \min_{\omega \models \dot{w}\dot{n}_w\dot{p}_w} \left\{ \sum_V \kappa_V(V(\omega)|pa(V)(\omega)) \right\} \\ &= \min_{\omega \models \dot{w}\dot{n}_w\dot{p}_w} \left\{ \begin{array}{l} \kappa_W(\dot{w}|\dot{p}_w) + \\ \sum_{V \in nd(W) \cup pa(W)} \kappa_V(V(\omega)|pa(V)(\omega)) + \\ \sum_{V \in desc(W)} \kappa_V(V(\omega)|pa(V)(\omega)). \end{array} \right\} \end{aligned}$$

The first sum here is fixed by the chosen configuration $\dot{n}_w, \dot{p}_w$, the min over the second sum is 0, as discussed in the proof of theorem 1:

$$\sum_{V \in nd(W) \cup pa(W)} \kappa_V(V(\omega)|pa(V)(\omega)) =: Const(\dot{n}_w, \dot{p}_w),$$

$$\min_{\omega \models \dot{w}\dot{n}_w\dot{p}_w} \left\{ \sum_{V \in desc(W)} \kappa_V(V(\omega)|pa(V)(\omega)) \right\} = 0,$$

hence $\kappa(\dot{w}\dot{n}_w\dot{p}_w) = \kappa_W(\dot{w}|\dot{p}_w) + Const(\dot{n}_w, \dot{p}_w)$. For the right hand side, we obtain similarly

$$\begin{aligned}
\kappa(\dot{n}_w\dot{p}_w) &= \min_{\omega \models \dot{n}_w\dot{p}_w} \left\{ \sum_V \kappa_V(V(\omega)|pa(V)(\omega)) \right\} \\
&= \min_{\omega \models \dot{n}_w\dot{p}_w} \left\{ \begin{array}{l} \kappa_W(W(\omega)|\dot{p}_w) + \\ \displaystyle\sum_{V \in nd(W) \cup pa(W)} \kappa_V(V(\omega)|pa(V)(\omega)) + \\ \displaystyle\sum_{V \in desc(W)} \kappa_V(V(\omega)|pa(V)(\omega)). \end{array} \right\} \\
&= Const(\dot{n}_w, \dot{p}_w) + \min_{\omega \models \dot{n}_w\dot{p}_w} \left\{ \begin{array}{l} \kappa_W(W(\omega)|\dot{p}_w) + \\ \displaystyle\sum_{V \in desc(W)} \kappa_V(V(\omega)|pa(V)(\omega)) \end{array} \right\} \\
&= Const(\dot{n}_w, \dot{p}_w) + \min_{\omega \models \dot{n}_w\dot{p}_w} \left\{ \kappa_W(W(\omega)|\dot{p}_w) \right\} = Const(\dot{n}_w, \dot{p}_w),
\end{aligned}$$

since $\min_{\omega \models \dot{n}_w\dot{p}_w} \left\{ \sum_{V \in desc(W)} \kappa_V(V(\omega)|pa(V)(\omega)) \right\} = 0$, independently of W having the value w or $\overline{w}$, and one of $\kappa_W(w|\dot{p}_w), \kappa_W(\overline{w}|\dot{p}_w)$ again must be 0.

Hence $\kappa(\dot{w}\dot{n}_w\dot{p}_w) = \kappa_W(\dot{w}|\dot{p}_w) + Const(\dot{n}_w, \dot{p}_w) = \kappa_W(\dot{w}|\dot{p}_w) + \kappa(\dot{n}_w\dot{p}_w)$, and this completes the proof. □

6 Filling in gaps by intensional combination

OCF-networks and stratifications are most valuable concepts for practical applications of the ranking framework as they help to cut down the complexity of full semantical information. However, one often has also to struggle with the problem of incomplete information, that is, only some (conditional) relationships between variables can be expressed with sufficient reliability. Typically, experts are quite certain about stating relationships between variables and each of its causes, or between variables and special configurations of its parents. In these cases, we first have to fill in missing values in the local ranking tables by somehow exploiting the partial explicit information, before we can apply the OCF-networks approach.

Hence we aim at calculating missing lines in the local table of ranking values $\kappa_V(V|pa(V))$ exploiting the available knowledge as well as possible and use inductive, intensional inference mechanisms like c-representations and System Z^+ on local knowledge. From these local ranking functions, we can easily read the missing tabular values for V and fill up the complete local tables. More precisely, the procedure for filling in missing values in the ranking tables is as follows:

Let a DAG Γ over Σ be given, and for each $V \in \Sigma$, let $\mathcal{R}_V$ be a local conditional knowledge base containing statements of the form $(\dot{v}|A)[m]$ where A is a formula involving only the parents of V. For example, $\mathcal{R}_V$ might have the form $\mathcal{R}_V = \{(\dot{v}|\dot{v}_i)[m_{\dot{v}_i}] | V_i \in pa(V)\}$.

In cases where $\mathcal{R}_V$ is not a complete conditional ranking table, do the following:

1. Consider $\mathcal{R}_V$ as a knowledge base over $\Sigma' = \{V\} \cup pa(V)$.
2. Compute an OCF κ_V over Σ' from $\mathcal{R}_V$ by using an inductive conditional reasoning method, like System Z+ or c-representations (cf. Section 4).
3. Compute from κ_V complete ranking tables $\kappa_V(V|pa(V))$ for every configuration of V and $pa(V)$.

Example 5. As an illustration, we modify an example from [3,4] that extends the example given in [12] by considering more complex information. A car starts ($S = s$) if the battery is charged ($B = b$) and the fuel tank is full ($F = f$). If either the battery is discharged ($B = \overline{b}$) or the fuel tank is empty ($F = \overline{f}$), the car does not start ($S = \overline{s}$); additionally, if, for some reason, the headlights have been left switched on overnight ($H = h$), the battery is discharged. We assume to know that it is very implausible to have left the headlights switched on ($\kappa_H(h) = 15$) and usually the tank is not empty ($\kappa_F(\overline{f}) = 10$). We also know that if the headlights have been switched on overnight, the battery is plausibly discharged ($\kappa_B(b|h) = 4$) but if the headlights have been switched off, the battery usually is charged ($\kappa_B(\overline{b}|\overline{h}) = 8$). Unfortunately, we are unaware of many ranking values at vertex S, in fact we just know that it is highly implausible that cars with an discharged battery and a full fueltank or cars with an empty fueltank but charged battery can be started ($\kappa_S(s|\overline{b}f) = 11$, $\kappa_S(s|b\overline{f}) = 13$). However, we know that it is highly implausible for a car with an empty battery to start, that is $\kappa_S(s|\overline{b}) = 12$, and even less plausible that a car without any fuel will start, that is $\kappa_S(s|\overline{f}) = 15$. On the other hand a car with a loaded battery usually starts $\kappa_S(\overline{s}|b) = 2$ and a car with a full fueltank should start, too, $\kappa_S(\overline{s}|f) = 1$. The OCF-network to this situation is shown in Figure 3, the local knowledge base from the joint information regarding S is

$$\mathcal{R}^*_S = \left\{ \begin{array}{l} r_1 = (s|b)[2], \quad r_2 = (\overline{s}|\overline{b})[12], \quad r_3 = (s|f)[1], \\ r_4 = (\overline{s}|\overline{f})[15], r_5 = (\overline{s}|b\overline{f})[13], r_6 = (\overline{s}|\overline{b}f)[11] \end{array} \right\}.$$

In this situation, we search for a local ranking function on S, B, F from which we can obtain the ranks of the vertices given all its parents. This can be achieved by using inductive conditional reasoning, i.e., by applying the methods presented in section 4. Note that, in constrast to [12], the local available knowledge at vertex S contains a more complex mixture of information regarding S. Some information is specialised enough to provide antries for the local table, but still the table has to be vompelted by using the the more general information for inductive reasoning First, we apply System Z^+. We compute the partition $\mathcal{R}_0$ of tolerated conditionals for this approach and find that $\mathcal{R}_0$ consists of the conditionals r_1, r_2, r_3 and r_4.

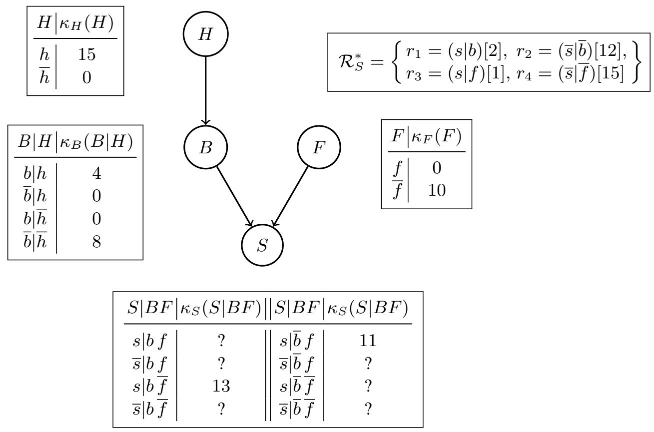


Fig. 3. Problem description of the car Example 5.

Table 1. Verification/falsification behaviour of configurations over the variables $\{B, F, S\}$ given the local car start knowledge base from Figure 3.

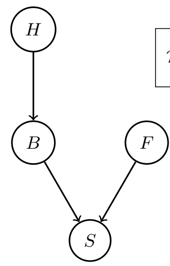
BFS	*verifies*	*falsifies*	$\kappa_S^Z(BFS)$	$\kappa_S^Z(BF)$	$\kappa_S^Z(S\|BF)$	$\kappa_S^c(BFS)$	$\kappa_S^c(BF)$	$\kappa_S^c(S\|BF)$
$b\,f\,s$	r_1, r_3	—	0	0	0	0	0	0
$b\,f\,\overline{s}$	—	r_1, r_3	2	0	2	3	0	3
$b\,\overline{f}\,s$	r_1	r_4, r_5	16	2	14	15	2	13
$b\,\overline{f}\,\overline{s}$	r_4, r_5	r_1	2	2	0	2	2	0
$\overline{b}\,f\,s$	r_3	r_2, r_6	13	1	12	12	1	11
$\overline{b}\,f\,\overline{s}$	r_2, r_6	r_3	1	1	0	1	1	0
$\overline{b}\,\overline{f}\,s$	—	r_2, r_4	15	0	15	27	0	27
$\overline{b}\,\overline{f}\,\overline{s}$	r_2, r_4	—	0	0	0	0	0	0

Therefore we can assign to each conditional in $\mathcal{R}_0$ the Z-value given as firmness in $\mathcal{R}_S$ and set $Z(r_1) = 2$, $Z(r_2) = 12$, $Z(r_3) = 1$ and $Z(r_4) = 15$. In the next steps of the algorithm we add, with a minimal world $b\overline{f}\,\overline{s}$, r_5 to $\mathcal{R}_0$ giving it the Z-value of $Z(r_5) = \kappa_Z^*(b\overline{f}\,\overline{s}) + 13 = 16$, followed by r_6 with a minimal world $\overline{b}fs$ and a Z-value of $Z(r_6) = \kappa_Z^*(\overline{b}f\overline{s}) + 11 = 13$.

We then set up a table indicating verification/falsification of the conditionals in $\mathcal{R}_S$ for each configuration of the local variables B, F and S, and associate with them the ranks according to formula (1). So we obtain the local ranking function $\kappa_S^Z(BFS)$ shown in Table 1. This table also proves useful to set up the inequalities needed to calculate a c-representation of the knowledge base according to inequation (2). Here we obtain $\kappa_1^- \geq 2 + \min\{0, \kappa_4^- + \kappa_5^-\} - \min\{0, \kappa_3^-\} = 2$, $\kappa_2^- \geq 12 + \min\{0, \kappa_3^-\} - \min\{\kappa_6^-, \kappa_4^-\}$, $\kappa_3^- \geq 1 + \min\{0, \kappa_2^- + \kappa_6^-\} - \min\{0, \kappa_1^-\} = 1$, $\kappa_4^- \geq 15 + \min\{0, \kappa_1^-\} - \min\{\kappa_5^-, \kappa_2^-\}$, $\kappa_5^- \geq 13 + \min\{\kappa_1^-\} - \min\{\kappa_4^-\}$ and $\kappa_6^- \geq 11 + \min\{\kappa_3^-\} - \min\{\kappa_2^-\}$. We choose one and set $\kappa_5^- = 0$ and $\kappa_6^- = 0$ and by this we get $\kappa_2^- = 12$ and $\kappa_4^- = 15$. By this we get a c-representation $\kappa_S^c(BFS)$ also shown Table 1.

H	$\kappa_H(H)$
h	15
$\overline{h}$	0

$B\|H$	$\kappa_B(B\|H)$
$b\|h$	4
$\overline{b}\|h$	0
$b\|\overline{h}$	0
$\overline{b}\|\overline{h}$	8



$$\mathcal{R}_S = \left\{ \begin{array}{l} r_1 = (s|b)[2],\ r_2 = (\overline{s}|\overline{b})[12], \\ r_3 = (s|f)[1],\ r_4 = (\overline{s}|\overline{f})[15] \end{array} \right\}$$

F	$\kappa_F(F)$
f	0
$\overline{f}$	10

$S\|BF$	$\kappa_S^Z(S\|BF)$	$\kappa_S^c(S\|BF)$	$S\|BF$	$\kappa_S^Z(S\|BF)$	$\kappa_S^c(S\|BF)$
$s\|b\,f$	0	0	$s\|\overline{b}\,f$	12	11
$\overline{s}\|b\,f$	2	3	$\overline{s}\|\overline{b}\,f$	0	0
$s\|b\,\overline{f}$	14	13	$s\|\overline{b}\,\overline{f}$	15	27
$\overline{s}\|b\,\overline{f}$	0	0	$\overline{s}\|\overline{b}\,\overline{f}$	0	0

Fig. 4. Solution for the car starting problem in Example 5.

Using this c-representation we can distinguish here between the configurations $b\,f\,\overline{s}$ and $b\,\overline{f}\,\overline{s}$ whereas this is not the case is we use the values calculated with System Z^+. With the determined values, we complete the local conditional ranking table $(S|BF)$ by calculating $\kappa_S(\dot{s}|\dot{b}\dot{f}) = \kappa_S(\dot{b}\dot{f}\dot{s}) - \kappa_S(\dot{b}\dot{f})$ for either one of the approaches shown in Table 1, too, and complete the graph from Fig. 3 to the OCF-network shown in Fig. 4.

Conclusion

In this paper we investigated Bayesian-style networks annotated with Spohn's ordinal conditional functions in place of probabilities, where values at some of the network's vertices are not specified. We demonstrated that well-known inductive reasoning approaches like System Z^+ as well as c-representations are capable of filling up the missing values with respect to the semantical structure of the problem. This application-oriented result helps us accomplishing the goal to allow the user of a system based on an OCF-network to specify her knowledge in an appropriate way and still rely on network techniques, leaving the technical details regarding local tables to the mentioned inference mechanisms. Previously to this, we examined whether formal properties of Bayesian networks are valid for OCF-networks, as well. More precisely, we proved that the local Markov property is valid for OCF-networks and showed that the global rankings of the stratified OCF coincides with the local rankings in the tables. In our continuing work, we explore these ideas for efficient implementations of OCF-based knowledge representation.

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