# Logic and XML

München

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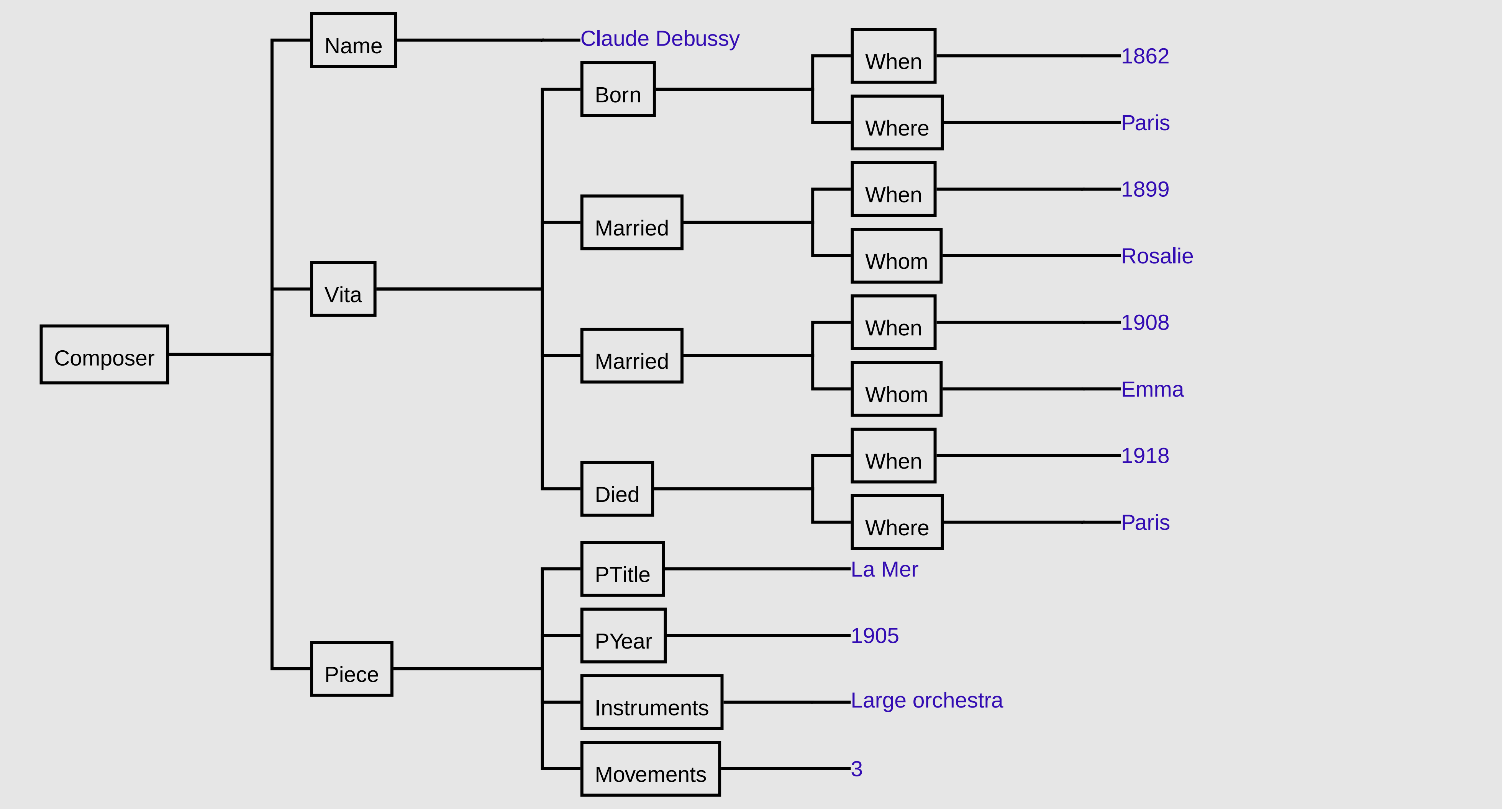
## Contents

# XML

## Example Tree



# XML Processing

| Four important kinds of XML processing ........................ | and their languages |
| --- | --- |
| **Validation**<br>Check whether an XML document is of a given type | `DTD`, `XML Schema` |
| **Navigation**<br>Select a set of positions in an XML document | `XPath` |
| **Querying**<br>Extract information from an XML document | `XQuery` |
| **Transformation**<br>Construct a new XML document from a given one | `XSLT` |

# Validation: DTD

## Example document

⟨Composer⟩
  ⟨Name⟩ Claude Debussy ⟨/Name⟩
  ⟨Vita⟩ ⟨Born⟩
      ⟨When⟩ August 22, 1862 ⟨/When⟩
      ⟨Where⟩ Paris ⟨/Where⟩
    ⟨/Born⟩ ⟨Married⟩
      ⟨When⟩ October 1899 ⟨/When⟩
      ⟨Whom⟩ Rosalie ⟨/Whom⟩
    ⟨/Married⟩ ⟨Married⟩
      ⟨When⟩ January 1908 ⟨/When⟩
      ⟨Whom⟩ Emma ⟨/Whom⟩
    ⟨/Married⟩ ⟨Died⟩
      ⟨When⟩ March 25, 1918 ⟨/When⟩
      ⟨Where⟩ Paris ⟨/Where⟩
    ⟨/Died⟩
  ⟨/Vita⟩
  ⟨Piece⟩
    ⟨PTitle⟩ La Mer ⟨/PTitle⟩
    ⟨PYear⟩ 1905 ⟨/PYear⟩
    ⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
    ⟨Movements⟩ 3 ⟨/Movements⟩
  ⟨/Piece⟩
⟨/Composer⟩
  ...

## DTD

DTDs describe types of XML documents

## Example

```
<!DOCTYPE Composers [
    <!ELEMENT Composers (Composer*)>
    <!ELEMENT Composer (Name, Vita, Piece*)>
    <!ELEMENT Vita (Born, Married*, Died?)>
    <!ELEMENT Born (When, Where)>
    <!ELEMENT Married (When, Whom)>
    <!ELEMENT Died (When, Where)>
    <!ELEMENT Piece (PTitle, PYear,
         Instruments, Movements)>
]>
```

# Navigation: XPath

## Example document

⟨Composer⟩
⟨Name⟩ Claude Debussy ⟨/Name⟩
⟨Vita⟩ ⟨Born⟩
⟨When⟩ August 22, 1862 ⟨/When⟩
⟨Where⟩ Paris ⟨/Where⟩
⟨/Born⟩ ⟨Married⟩
⟨When⟩ October 1899 ⟨/When⟩
⟨Whom⟩ Rosalie ⟨/Whom⟩
⟨/Married⟩ ⟨Married⟩
⟨When⟩ January 1908 ⟨/When⟩
⟨Whom⟩ Emma ⟨/Whom⟩
⟨/Married⟩ ⟨Died⟩
⟨When⟩ March 25, 1918 ⟨/When⟩
⟨Where⟩ Paris ⟨/Where⟩
⟨/Died⟩
⟨/Vita⟩
⟨Piece⟩
⟨PTitle⟩ La Mer ⟨/PTitle⟩
⟨PYear⟩ 1905 ⟨/PYear⟩
⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
⟨Movements⟩ 3 ⟨/Movements⟩
⟨/Piece⟩
⟨/Composer⟩
...

## XPath

XPath expressions select sets of nodes of XML documents by specifying navigational patterns

## Example query

```
//Vita/Died/*
```

## Remark

XPath expressions define sets of nodes:

**node-selecting queries**

# Querying: XQuery

## Example document

⟨Composer⟩
⟨Name⟩ Claude Debussy ⟨/Name⟩
⟨Vita⟩ ⟨Born⟩
⟨When⟩ August 22, 1862 ⟨/When⟩
⟨Where⟩ Paris ⟨/Where⟩
⟨/Born⟩ ⟨Married⟩
⟨When⟩ October 1899 ⟨/When⟩
⟨Whom⟩ Rosalie ⟨/Whom⟩
⟨/Married⟩ ⟨Married⟩
⟨When⟩ January 1908 ⟨/When⟩
⟨Whom⟩ Emma ⟨/Whom⟩
⟨/Married⟩ ⟨Died⟩
⟨When⟩ March 25, 1918 ⟨/When⟩
⟨Where⟩ Paris ⟨/Where⟩
⟨/Died⟩
⟨/Vita⟩
⟨Piece⟩
⟨PTitle⟩ La Mer ⟨/PTitle⟩
⟨PYear⟩ 1905 ⟨/PYear⟩
⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
⟨Movements⟩ 3 ⟨/Movements⟩
⟨/Piece⟩
⟨/Composer⟩
...

## XQuery

XQuery is a full-fledged XML query language

### Example query

```
for $x in
doc('composers.xml')/Composer
where $x/Vita/Died/Where =
'Paris'
return $x/Name
```

### Result

⟨Name⟩ Claude Debussy ⟨/Name⟩
⟨Name⟩ Eric Satie ⟨/Name⟩
⟨Name⟩ Hector Berlioz ⟨/Name⟩
⟨Name⟩ Camille Saint-Saëns ⟨/Name⟩
⟨Name⟩ Frédéric Chopin ⟨/Name⟩
⟨Name⟩ Maurice Ravel ⟨/Name⟩
⟨Name⟩ Jim Morrison ⟨/Name⟩
⟨Name⟩ César Franck ⟨/Name⟩
⟨Name⟩ Gabriel Fauré ⟨/Name⟩
⟨Name⟩ George Bizet ⟨/Name⟩
...

# Transformation: XSLT

## Example document

```
⟨Composer⟩
  ⟨Name⟩ Claude Debussy ⟨/Name⟩
  ⟨Vita⟩ ⟨Born⟩
      ⟨When⟩ August 22, 1862 ⟨/When⟩
      ⟨Where⟩ Paris ⟨/Where⟩
    ⟨/Born⟩ ⟨Married⟩
      ⟨When⟩ October 1899 ⟨/When⟩
      ⟨Whom⟩ Rosalie⟨/Whom⟩
    ⟨/Married⟩ ⟨Married⟩
      ⟨When⟩ January 1908 ⟨/When⟩
      ⟨Whom⟩ Emma ⟨/Whom⟩
    ⟨/Married⟩ ⟨Died⟩
      ⟨When⟩ March 25, 1918 ⟨/When⟩
      ⟨Where⟩ Paris ⟨/Where⟩
    ⟨/Died⟩
  ⟨/Vita⟩
  ⟨Piece⟩
    ⟨PTitle⟩ La Mer ⟨/PTitle⟩
    ⟨PYear⟩ 1905 ⟨/PYear⟩
    ⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
    ⟨Movements⟩ 3 ⟨/Movements⟩
  ⟨/Piece⟩
⟨/Composer⟩
        ...
```

## XSLT

XSLT transforms documents by means of templates

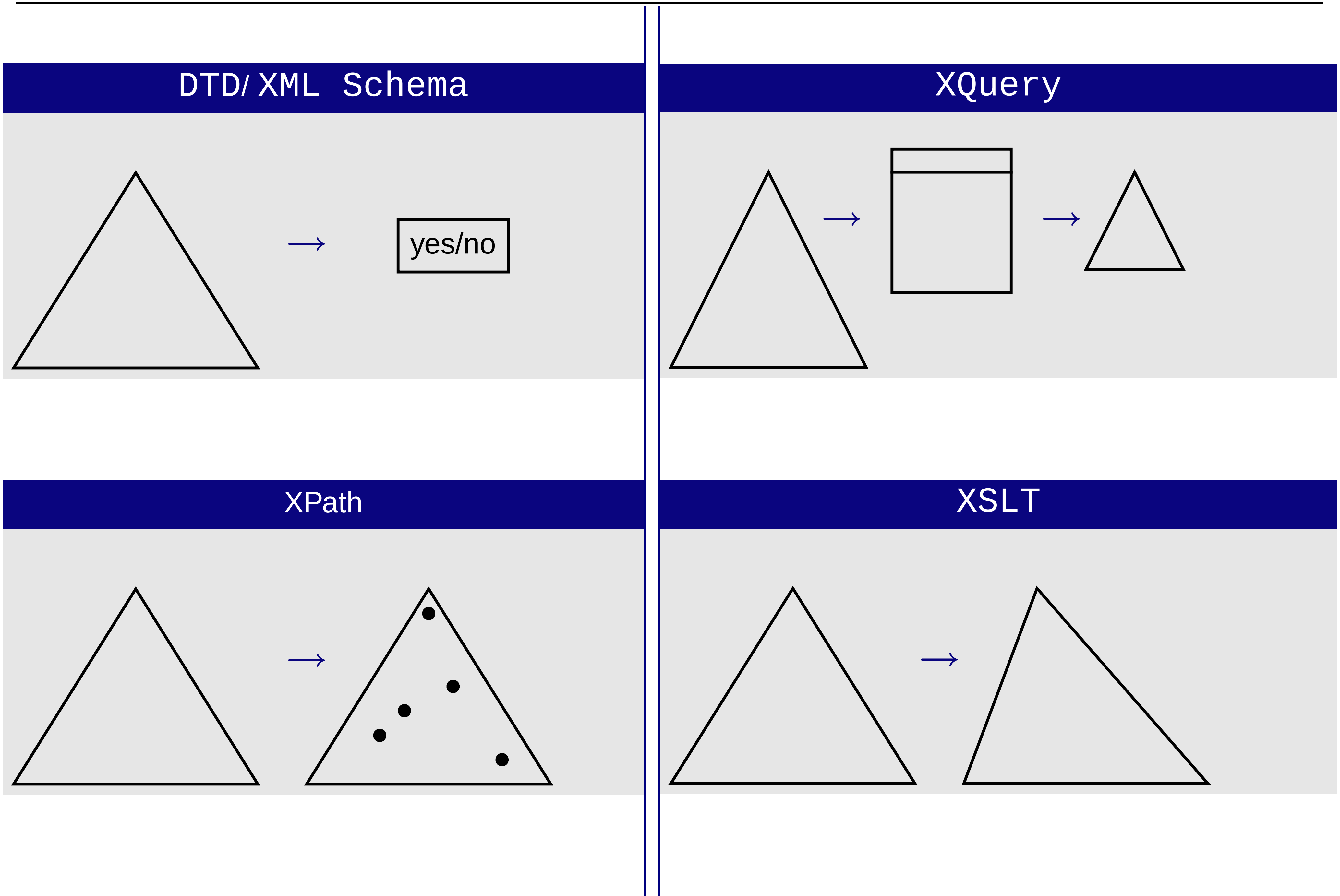
### Example

```
⟨xsl:template match=”Composer[Vita//Where=’Paris’]”⟩
  ⟨ParisComposer⟩
    ⟨xsl:copy-of select=“Name”/⟩
    ⟨xsl:copy-of select=“Vita/Born”/⟩
  ⟨/ParisComposer⟩
⟨/xsl:template⟩
```

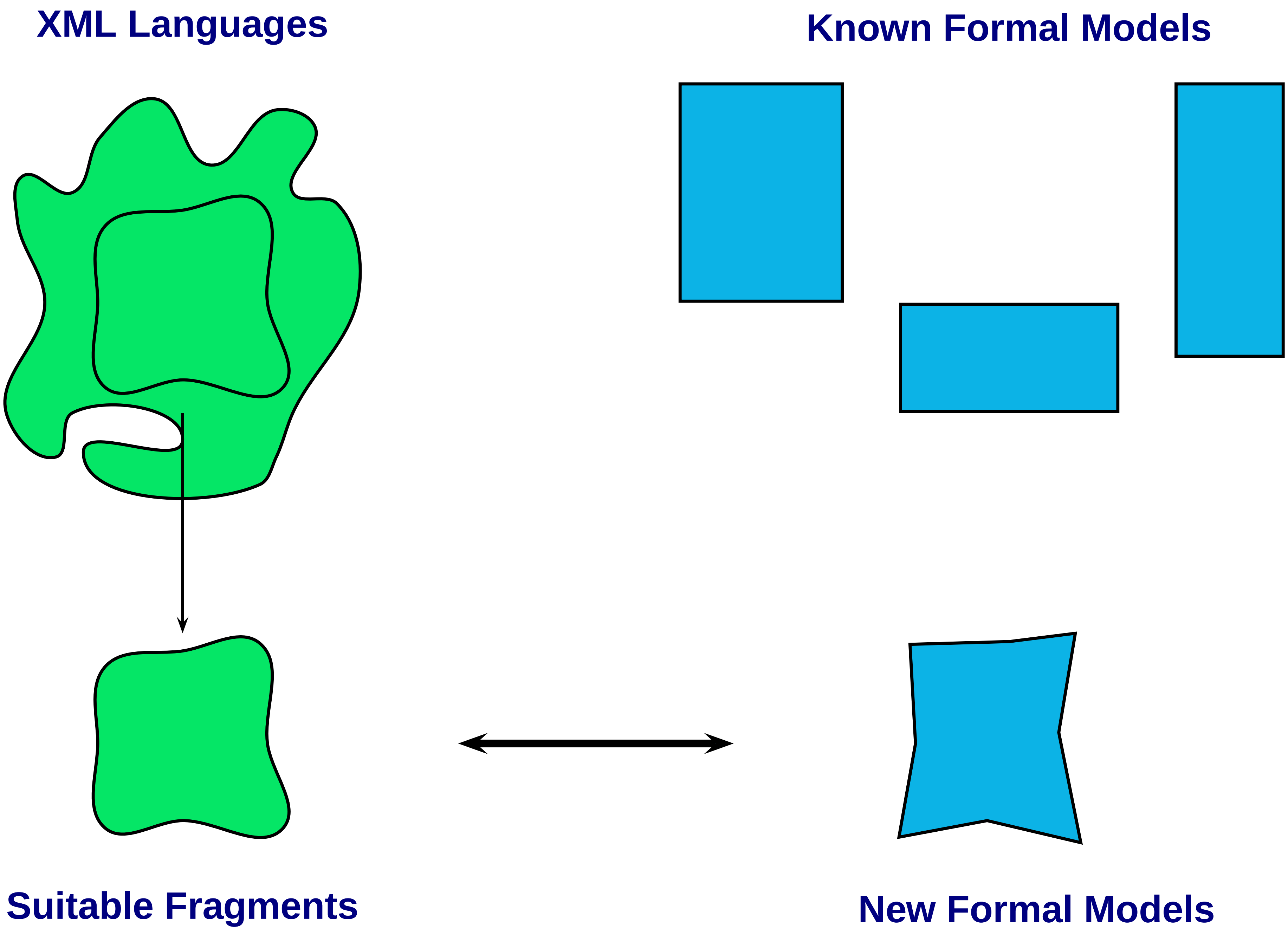
### Result

```
⟨ParisComposer⟩⟨Name⟩ Claude Debussy ⟨/Name⟩
  ⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩
    ⟨Where⟩ Paris ⟨/Where⟩
  ⟨/Born⟩⟨/ParisComposer⟩
⟨ParisComposer⟩ ⟨Name⟩ Frédéric Chopin ⟨/Name⟩
  ⟨Born⟩ ⟨When⟩ March 1, 1810 ⟨/When⟩
    ⟨Where⟩ Żelazowa ⟨/Where⟩
  ⟨/Born⟩⟨/ParisComposer⟩
⟨ParisComposer⟩ ⟨Name⟩ Camille Saint-Saëns ⟨/Name⟩
  ⟨Born⟩ ⟨When⟩ October 9, 1835 ⟨/When⟩
  ⟨Where⟩ Paris ⟨/Where⟩
⟨/Born⟩ ⟨/ParisComposer⟩
```

# A Schematic View



# The Big Picture



# Algorithmic Tasks

## Evaluation

### Evaluation (Combined)

**I:** Tree $t$, Query $q$

**O:** $q(t)$

### Evaluation (Data($q$))

**I:** Tree $t$

**O:** $q(t)$

### Incremental Eval. ($q$)

**I:** Tree $t$,

Changes of $t$

**O:** $q(t)$

## Static Analysis

### Satisfiability

**I:** Query $q$

**Q:** Is $q(t) \neq \emptyset$

for some $t$?

### Containment

**I:** Queries $q_1$,$q_2$

**Q:** Is always

$q_1(t) \subseteq q_2(t)$?

### Equivalence

**I:** Queries $q_1$,$q_2$

**Q:** Is always

$q_1(t) = q_2(t)$?

### Type Checking

**I:** Types $d_1$,$d_2$,

Transformation $T$

**Q:** Does $t \models d_1$ imply

$T(t) \models d_2$?

### Type Inference

**I:** Types $d$,

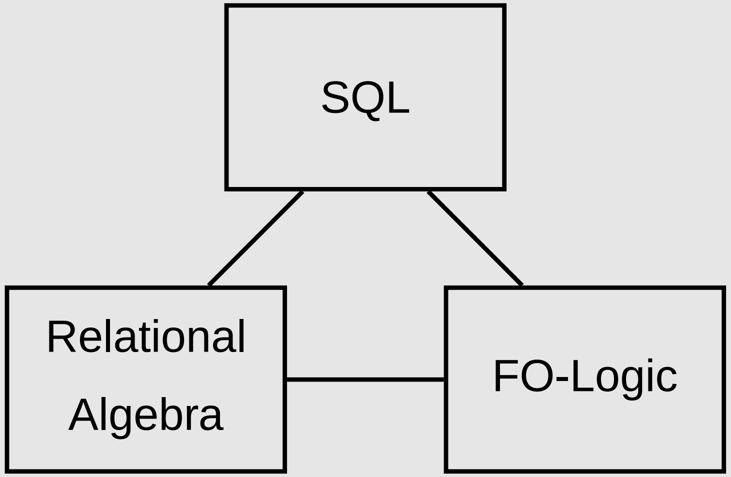
Transformation $T$

**O:** Type of

$\{T(t) \mid t \models d\}$

# A Look Back

## Relational Databases




## Basic Properties of these Formalisms

**SQL**

- Declarative, easy to use
- Queries, data definition, updates

**FO-logic**

- Formal framework for investigations
- Clear Semantics
- Expressive power well understood

**Relational Algebra**

- Operational model
- Flexible, optimizable
- Automatic translation

## Further Properties

- Satisfiability undecidable

$\longrightarrow$ Fragments like conjunctive queries

---

- Evaluation for conjunctive queries **NP**-hard

$\longrightarrow$ but works well in practice

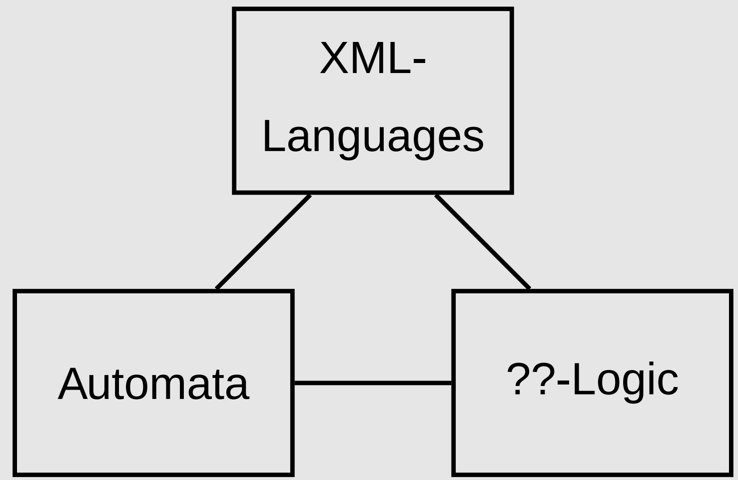
---

- SQL can count and group

$\longrightarrow$ Can be added to FO

[Hella et al. 01]

## Goal



Contents

# XML as Tree

## Example Document

⟨a⟩ ⟨b⟩

⟨d⟩12⟨/d⟩ ⟨e⟩22⟨/e⟩

⟨/b⟩

⟨b⟩ ⟨d⟩4⟨/d⟩ ⟨/b⟩

⟨b⟩ ⟨d⟩11⟨/d⟩

⟨e⟩8⟨/e⟩18⟨e⟩5⟨/e⟩

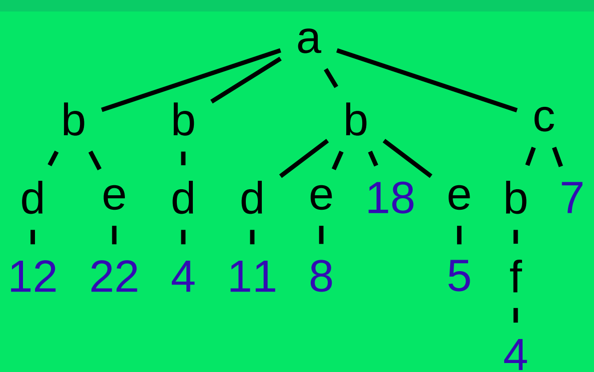
⟨/b⟩

⟨c⟩ ⟨b⟩ ⟨f⟩2⟨/f⟩

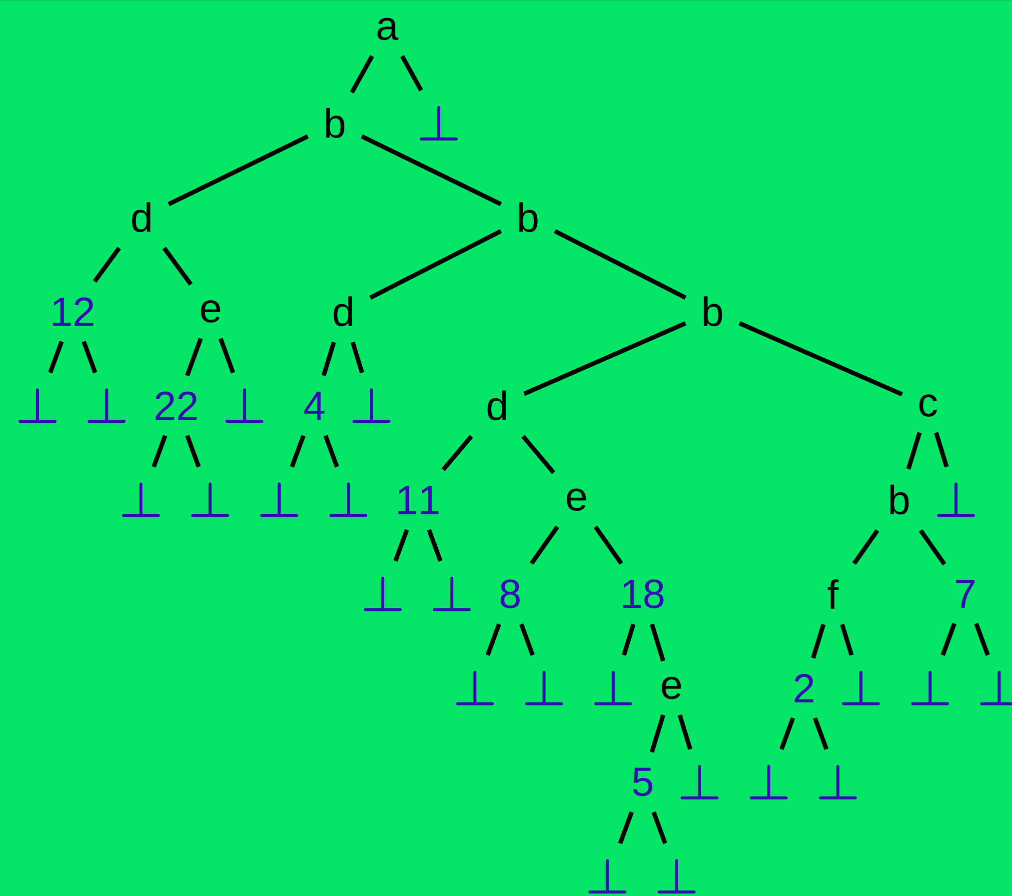
⟨/b⟩7⟨/c⟩

⟨/a⟩

## ...as Unranked Tree




## ...as Binary Tree

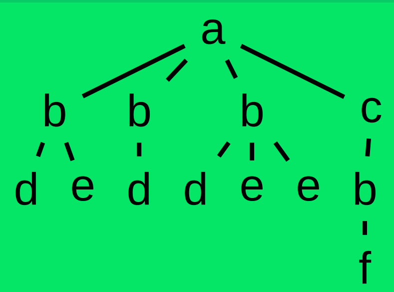


# XML Trees as Finite Models

## Data Values

For many investigations, data values can and have to be ignored

### Example




## Labels

- Usually, XML trees are modeled as labeled trees over a finite alphabet
- For schema languages this is ok
- For queries, the actual alphabet might depend on the query
- Unary predicates on data values can be also modeled this way

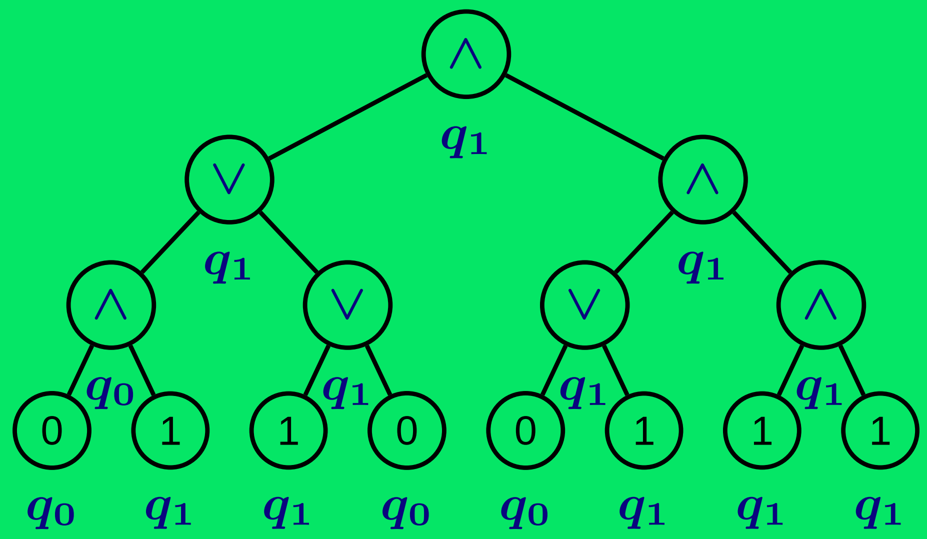
## Signatures

- There are various ways to represent unordered trees as finite relational models
- Possible relations
  - Local orders: $\longrightarrow, \downarrow$ $\longleftarrow, \uparrow$ $\swarrow$
  - Their transitive closures: $\longrightarrow^+, \downarrow^+, \longleftarrow^+, \uparrow^+$
  - Reflexive and transitive: $\longrightarrow^*, \downarrow^*, \longleftarrow^*, \uparrow^*$
  - Document order: $\uparrow^*\longrightarrow\downarrow^*$
- Frequent combinations:
  - $\longrightarrow, \downarrow$
  - $\longrightarrow^+, \downarrow^+$
  - $\longrightarrow, \downarrow, \longrightarrow^+, \downarrow^+$
  - $\longrightarrow, \swarrow$

# Contents

# Automata for Ranked Trees

## Bottom-up Automaton for (Tree-) Boolean Circuits




### Idea

Two states: $q_0, q_1$

$q_1 \equiv$ subtree evaluates to 1

### Transitions

$\delta(\wedge, q_1) = \{(q_1, q_1)\}$

$\delta(\wedge, q_0) = \{(q_0, q_1), (q_1, q_0), (q_0, q_0)\}$

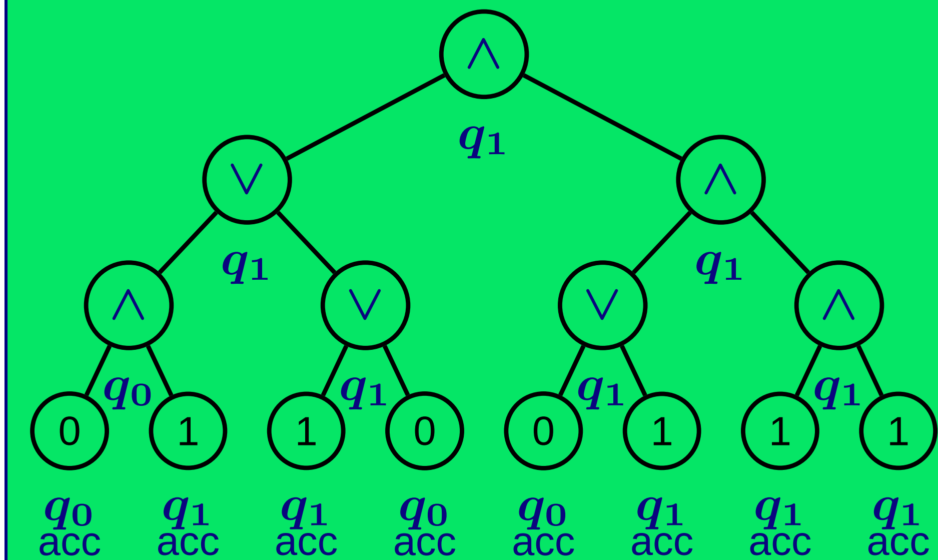
$\delta(\vee, q_1) = \{(q_0, q_1), (q_1, q_0), (q_1, q_1)\}$

$\delta(\vee, q_0) = \{(q_0, q_0)\}$

$\delta(0, q_0) = \{\epsilon\}; \delta(0, q_1) = \emptyset$

$\delta(1, q_1) = \{\epsilon\}; \delta(1, q_0) = \emptyset$

## Top-down Automaton for Boolean Circuits




### Idea

Three states: $q_0, q_1$, acc

$q_1 \equiv$ subtree will be 1

### Transitions

$\delta(\wedge, q_1) = \{(q_1, q_1)\}$

$\delta(\wedge, q_0) = \{(q_0, q_1), (q_1, q_0), (q_0, q_0)\}$

$\delta(\vee, q_1) = \{(q_0, q_1), (q_1, q_0), (q_1, q_1)\}$

$\delta(\vee, q_0) = \{(q_0, q_0)\}$

$\delta(0, q_0) = \{\text{acc}\}; \delta(0, q_1) = \emptyset$

$\delta(1, q_1) = \{\text{acc}\}; \delta(1, q_0) = \emptyset$

# Regular Tree Languages

## Definition

A bottom-up automaton is deterministic if for each $a$ and $p \neq q$:

$$\delta(a,p) \cap \delta(a,q) = \emptyset$$

## Theorem

The following are equivalent for a tree language $L$:

(a) $L$ is accepted by a nondeterministic bottom-up automaton

(b) $L$ is accepted by a deterministic bottom-up automaton

(c) $L$ is accepted by a nondeterministic top-down automaton

## Proof

(a) $\Longrightarrow$ (b): Powerset construction

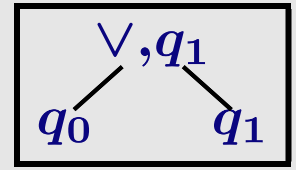
(a) $\Longleftrightarrow$ (c): Just the same thing, viewed in two different ways

## Definition

Such an $L$ is called regular

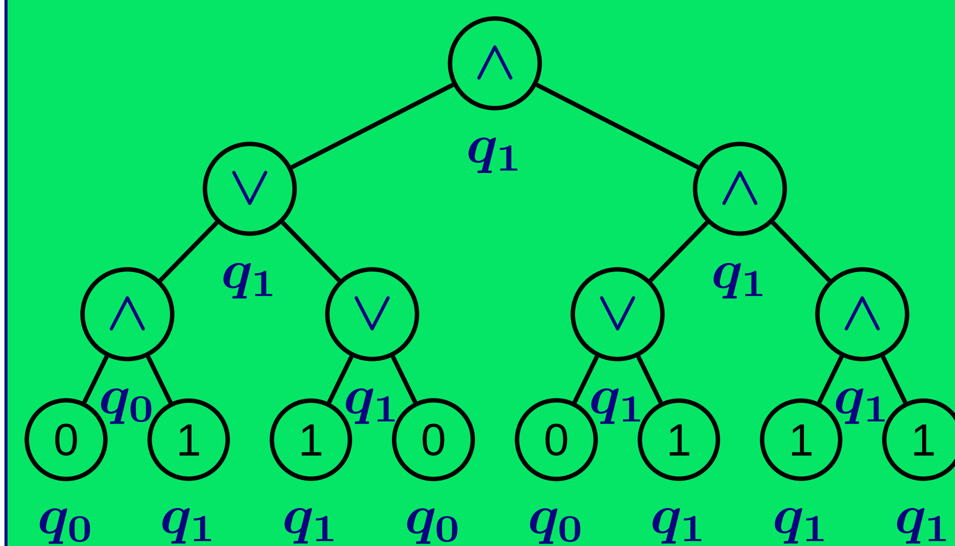
## Observation

- $(q_0,q_1) \in \delta(\vee,q_1)$ can be interpreted as an allowed pattern:




- A tree is accepted, iff there is a labelling with states such that
  - all local patterns are allowed
  - the root is labelled with $q_1$

## Example



# MSO and Regular Tree Languages

## Definition: MSO logic

- **MSO-formulas** talk about
  - node labels ($Q_0$,$Q_1$,$Q_\wedge$,$Q_\vee$)
  - Children and neighbors: $\rightarrow$, $\downarrow$
  - the root of the tree (root)
- **First-order-variables** represent nodes
- **Monadic second-order** (MSO) variables represent sets of nodes

## Remark

Exact signature does not matter

## Example: Boolean Circuits

$$\exists X \; X(\text{root}) \wedge \; \forall x$$
$$(Q_0(x) \rightarrow \neg X(x)) \wedge$$
$$((Q_\wedge(x) \wedge X(x)) \rightarrow (\forall y[(x \downarrow y) \rightarrow X(y)])) \wedge$$
$$((Q_\vee(x) \wedge X(x)) \rightarrow (\exists y[(x \downarrow y) \wedge X(y)]))$$

## Theorem (Doner 70; Thatcher, Wright 68)

On ranked trees:

MSO $\equiv$ Regular Tree Languages

## Proof idea

**Automata** $\Rightarrow$ **MSO**:
Formula expresses that there exists a correct tiling

**MSO** $\Rightarrow$ **Automata**: more involved

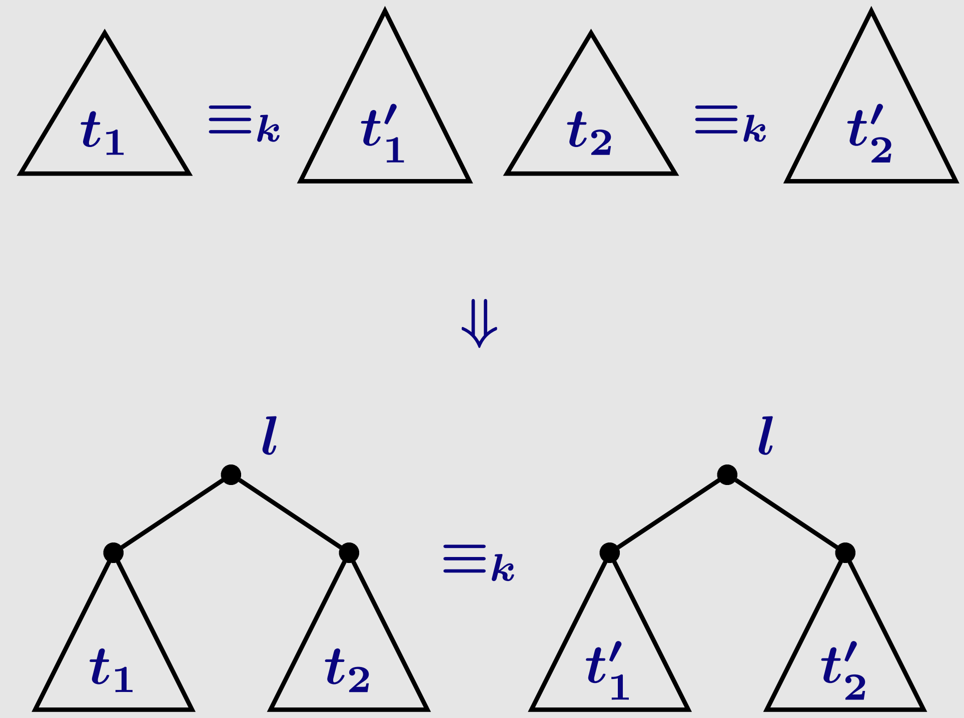
Basic idea:
Automaton computes for each node $v$ the set of formulas which hold in the subtree rooted at $v$

# MSO and Regular Tree Languages (cont.)

## MSO $\Longrightarrow$ Automata: more precisely

- Let $\varphi$ be an MSO-formula
  $k :=$ quantifier-depth of $\varphi$
- **$k$-type** of a tree $t :=$ (essentially)
  set of MSO-formulas $\psi$ of quantifier-depth $\leq k$ which hold in $t$
- **$t_1 \equiv_k t_2$**:
  $k$-type$(t_1) = k$-type$(t_2)$
- Automaton computes $k$-type of tree and concludes whether $\varphi$ holds

## Crucial fact

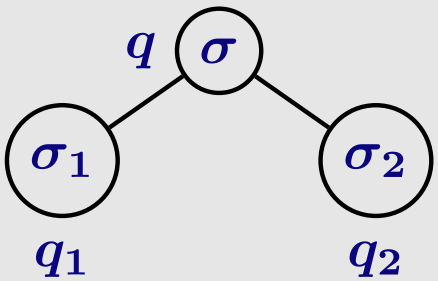


# From Ranked to Unranked Trees

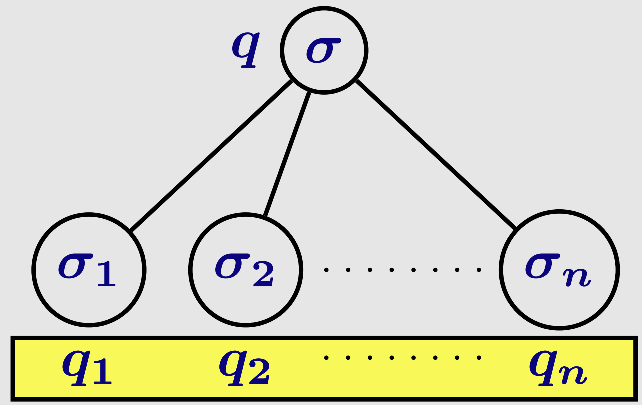
## Ranked trees

On ranked trees, transitions are described by finite sets:

$$\delta(\sigma,q) = \{(q_1,q_2),(q_3,q_4),\ldots\}$$




## Unranked trees




$\in \delta(\sigma,q)$?

- For unranked trees, $\delta(\sigma,q)$ is a regular language
- $\delta(\sigma,q)$ can be specified by regular expression or finite string automaton [Brggemann-Klein, Murata, Wood 2001]

# Regular Tree Languages and Complexity

## Theorem

The following are equivalent for a set $L$ of unranked trees:

(a) $L$ is accepted by a nondeterministic bottom-up automaton

(b) $L$ is accepted by a deterministic bottom-up automaton

(c) $L$ is accepted by a nondeterministic top-down automaton

(d) $L$ is characterized by an MSO-formula

## Definition

Again: such an $L$ is called **regular**

## Complexity issues for MSO on trees

**Data Complexity:**

Query evaluation is possible in time $O(|t|)$

**Combined Complexity:**

- Query evaluation is complete for **PSPACE**
- Query evaluation is possible in time $f(|\varphi|)|t|$,

  where $f$ is $\sim 2^{2^{\cdot^{\cdot^{\cdot^{2^{2^{|\varphi|}}}}}}} \Big\} |\varphi|$
- No elementary $f$ possible unless **P** $=$ **NP** [Frick, Grohe 2002]
- Satisfiability: $f(|\varphi|)$ (same $f$)
- In practice much better: MONA [Klarlund et al.]

Contents

# DTDs and their Weakness

## DTDs

- DTDs are essentially generalized context-free grammars

→ [Berstel,Boasson 00] provide characterizations

### Example

```
<!DOCTYPE Composers [
    <!ELEMENT Composers (Composer*)>
    <!ELEMENT Composer (Name, Vita, Piece*)>
    <!ELEMENT Vita (Born, Married*, Died?)>
    <!ELEMENT Born (When, Where)>
    <!ELEMENT Married (When, Whom)>
    <!ELEMENT Died (When, Where)>
    <!ELEMENT Piece (PTitle, PYear,
         Instruments, Movements)>
]>
```

## Weakness of DTDs

- Elements with the same name may have different structure in different contexts

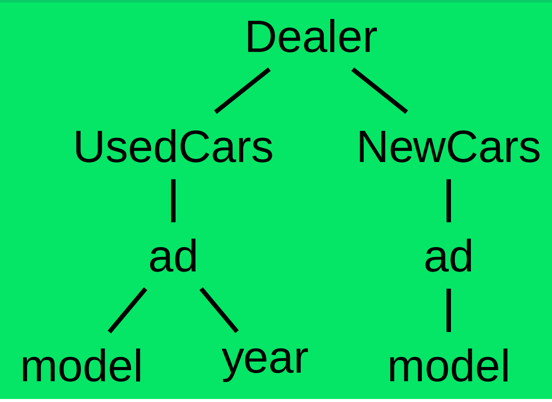
→ It would be nice to have types for elements

→ **Extended DTDs**

### A classical example

```
<!DOCTYPE Dealer [
    <!ELEMENT Dealer (UsedCars NewCars)>
    <!ELEMENT UsedCars (ad*)>
    <!ELEMENT NewCars (ad*)>
    <!ELEMENT ad ((model, year) | model)>
]>
```

### Intention



# Extended DTDs

**Definition [Papakonstantinou, Vianu 2000]**

An **extended DTD** (EDTD) over alphabet $\Sigma$ is a pair $(d, \mu)$, where

- $d$ is a DTD over the alphabet $\Sigma'$ of **types**
- $\mu : \Sigma' \longrightarrow \Sigma$ maps types to tag names

**Example**

| | |
|---|---|
| Dealer $\longrightarrow$ UsedCars NewCars | $\mu$(Dealer) $=$ Dealer |
| UsedCars $\longrightarrow$ adUsed$^*$ | $\mu$(UsedCars) $=$ UsedCars |
| NewCars $\longrightarrow$ adNew$^*$ | $\mu$(NewCars) $=$ NewCars |
| adUsed $\longrightarrow$ model year | $\mu$(adUsed) $=$ ad |
| adNew $\longrightarrow$ model | $\mu$(adNew) $=$ ad |

**Note**

Extended DTDs are often called *specialized DTDs*

**EDTD for Boolean circuits**

| | | |
|---|---|---|
| 1-AND | $\longrightarrow$ | (1-OR \| 1-AND \| 1-leaf)* |
| 1-OR | $\longrightarrow$ | $.^*$ (1-OR \| 1-AND \| 1-leaf) $.^*$ |
| 0-AND | $\longrightarrow$ | $.^*$ (0-OR \| 0-AND \| 0-leaf) $.^*$ |
| 0-OR | $\longrightarrow$ | (0-OR \| 0-AND \| 0-leaf)* |
| 1-leaf | $\longrightarrow$ | $\epsilon$ |
| 0-leaf | $\longrightarrow$ | $\epsilon$ |

| Tag | $\mu$(Tag) |
|---|---|
| 1-AND | AND |
| 0-AND | AND |
| 1-OR | OR |
| 0-OR | OR |
| 1-leaf | 1 |
| 0-leaf | 0 |

# Extended DTDs and MSO

### Observation

- A tree conforms to an extended DTD $(d,\mu)$ if there is a labeling of its nodes by types which is valid wrt. $d$
- This reminds us of something...

### Theorem

Extended DTDs capture exactly the regular tree languages

### Remarks

- Regular tree languages and MSO-logic are a convenient framework for the study of XML schema languages
- Practical languages as XML Schema usually correspond to subclasses
- Full MSO power: Relax NG
- What about queries?

## Contents

# Node-selecting Queries

## Question

Is there an equally robust class of node-selecting queries?

## Observations

- There is a simple way to define node selecting queries by monadic second-order formulas:
- Simply use one free variable: $\varphi(x)$
- Is there a corresponding automaton model?
- It is relatively easy to add node selection to nondeterministic bottom-up automata

## Definition

- **Nondetermistic node-selecting automata**
- Nondeterministic bottom-up automata plus select function:
$$s : Q \times \Sigma \to \{0,1\}$$
- Node $v$ is in result set for tree $t$ $:\iff$ there is an accepting computation on $t$ in which $v$ gets a state $q$ such that $s(q,\lambda(v)) = 1$

## Existential vs. universal semantics

- Existential semantics: a node is in the result if there exists an accepting run which selects it
- Universal semantics: a node is in the result if every accepting run selects it
- Both semantics define the same class of queries

## Theorem

A node selecting query is MSO-definable iff it is expressible by a nondeterministic bottom-up node selecting automaton

# Node-selecting queries (cont.)

**Theorem**

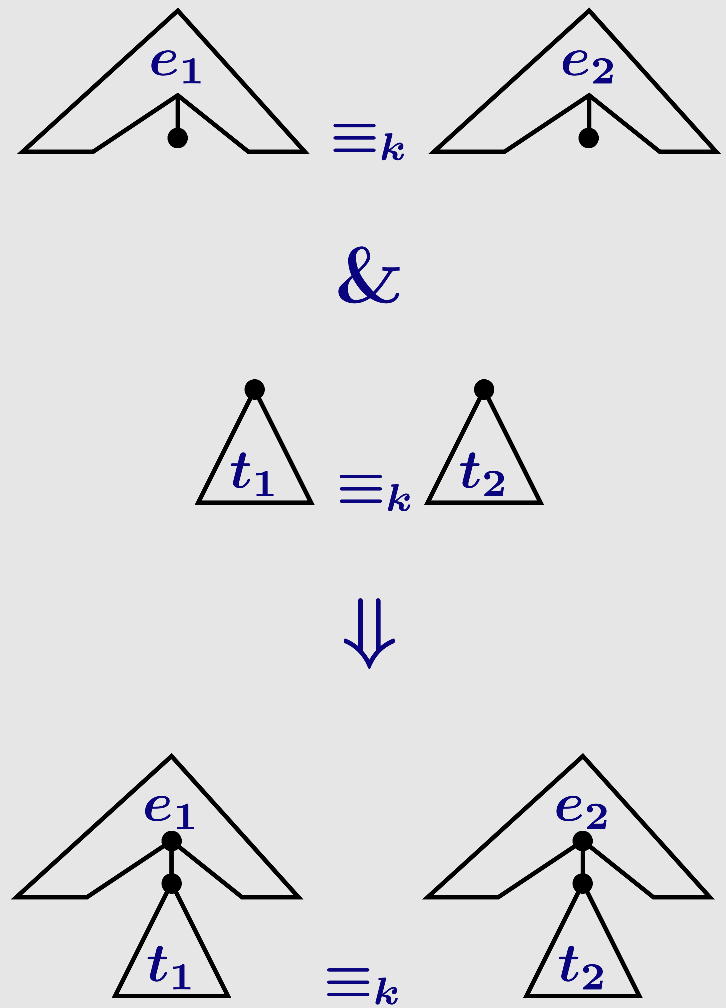
A node selecting query is MSO-definable iff it is expressible by a nondeterministic bottom-up node selecting automaton

**Proof idea**

- Given formula $\varphi(x)$ of quantifier-depth $k$ and tree $t$,
  for each node $v$ the automaton does the following:
  - Compute $k$-type of subtree at $v$
  - Guess $k$-type of "envelope tree" at $v$
  - Conclude whether $v$ is in the output
  - Check consistency upwards towards the root

$\Rightarrow$ one unique accepting run

**Crucial fact**



# Node-selecting queries (cont.)

## More query models

- Same combined complexity as for language recognition (**PSPACE**-complete)

$\longrightarrow$ query languages with better complexity properties needed

- Good candidate: Monadic Datalog [Gottlob, Koch 2002] and its restricted dialects like TMNF
- Further models:
  - Attributed Grammars [Neven, Van den Bussche 1998]
  - $\mu$-formulas [Neumann 1998]
  - Context Grammars [Neumann 1999]
  - Deterministic Node-Selecting Automata [Neven, Sch. 1999]

## Data complexity

- MSO node-selecting queries can be evaluated in two passes

  **First pass, bottom-up:** Compute the types of the subtrees

  **Second pass, top-down:** Compute the types of the envelopes and combines it with the subtree information

$\longrightarrow$ Can be implemented by a 2-pass pushdown document automaton which in its first pass attaches information to each node [Neumann, Seidl 1998; Koch 2003]

- In particular: Data complexity is linear time

## Contents

# XML Transformations and MSO-Logic

## Definition: XSLT TypeChecking

**Given:** DTDs $d_1$ and $d_2$

Transformation $T$

**Result:** Is $T(t) \models d_2$ for each document $t$ with $t \models d_1$?

## Questions

- Is XSLT TypeChecking decidable?
- What is the complexity?

## Outline

- Provide an automata model for XSLT transformations
- Show that the behaviour of these automata can be captured by MSO logic
- Use manipulation of regular tree languages to solve type checking problem
- [Milo,Suciu,Vianu 01]

# XSLT in more detail

**How XSLT roughly works**

**Templates:** ⟨`xsl:template name=TName match=pattern mode=MName`⟩

**Template application:**
⟨`xsl:apply-templates select=Expression mode=MName`⟩

**XSLT Processing** Whenever `xsl:apply-templates` is called at a node $v$ the following happens:

- Compute set $S(v)$ of nodes, reachable from $v$ via `Expression` (if `select` is not present, $S(v) =$ children of $v$)
- For each $w \in S(v)$ compute which templates can be applied to $w$:
  - $w$ has to match `pattern` of a template
  - the `mode` of the template has to be the same as the mode of `xsl:apply-templates`
- If no template matches, take the default template
- For each $w \in S(v)$ select the best template and apply it.

The process starts at the root of the tree

# XSLT: Example

## Example Transformation

*Remove everything below a $c$. Translate $a$ below $b$ into $d$*

## Example XSLT (Abbreviated)

⟨... match="a"⟩ ⟨a⟩ ⟨xsl:apply-templates⟩ ⟨/a⟩ ⟨/...⟩

⟨... match="a" mode="below"⟩ ⟨d⟩ ⟨xsl:apply-templates⟩ ⟨/d⟩ ⟨/...⟩

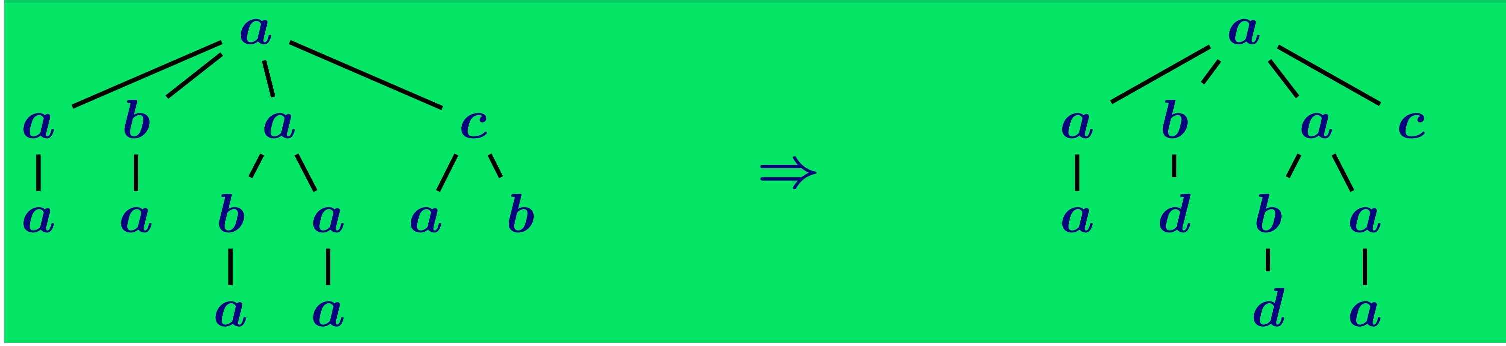
⟨... match="b"⟩ ⟨b⟩ ⟨xsl:apply-templates mode="below"⟩ ⟨/b⟩ ⟨/...⟩

⟨... match="b" mode="below"⟩ ⟨b⟩ ⟨xsl:apply-templates mode="below"⟩ ⟨/b⟩ ⟨/...⟩

⟨... match="c"⟩ ⟨c⟩ ⟨/c⟩ ⟨/...⟩

⟨... match="c" mode="below"⟩ ⟨c⟩ ⟨/c⟩ ⟨/...⟩

## Example Trees



# An automaton model for XSLT

**Definition: $k$-pebble Transducer**

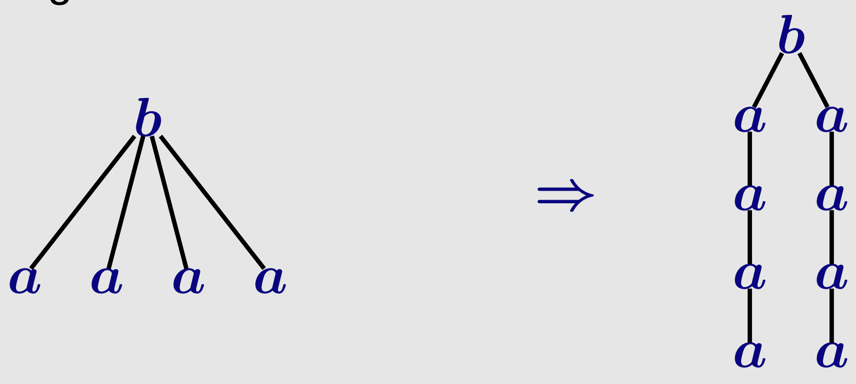
- Work on binary tree encodings of unranked trees
- Up to $k$ pebbles can be placed on the tree
- Only pebble with highest number (*current pebble*) can move, depending on state, number of pebbles, symbols under pebbles and incidence of pebbles
- Possible pebble movements:
  - stay
  - go to left child, right child or parent
  - lift current pebble
  - place new pebble on the root
- Nondeterminism allowed

  (Proof presented here: deterministic case)
- If current pebble stays it is possible to produce output:
  - a node with two (forthcoming) subtrees; in this case two independent subcomputations (*branches*) are started, which construct the left subtree and right subtree, respectively
  - a leaf; in this case the computation branch stops

# Back to the Typechecking Question

## Proof idea

- How can we check that $T(t) \in L(d_2)$, for each $t \in L(d_1)$?
- Obvious approach:
  - Compute $T(L(d_1))$
  - Check that $T(L(d_1)) \subseteq L(d_2)$
- Problem: $T(L(d_1))$ does not need to be regular:




- Better approach:
  - Compute $T^{-1}(\overline{L(d_2)})$
  - Check $L(d_1) \cap T^{-1}(\overline{L(d_2)}) = \emptyset$

## Definition: $k$-pebble acceptors

- Basically the same as $k$-pebble transducers
- Instead of output producing steps:
  - *accept*
  - branch into two independent subcomputations
- A tree is accepted if all subcomputations accept

## Main Steps of the Proof

(i) $T$ computed by $k$-pebble transducer

$$\Rightarrow T = T_1 \circ \cdots \circ T_{k+1}$$

with $0$-pebble transducers $T_i$

[Engelfriet, Maneth 03]

(ii) $L$ regular, $T_i$ $0$-pebble transducer

$$\Rightarrow$$

$T_i^{-1}(L)$ accepted by $0$-pebble acceptor

(iii) $0$-pebble acceptors only accept regular tree languages

# Step (ii)

**Lemma**

$L$ regular, $T_i$ $\mathbf{0}$-pebble transducer
$\Rightarrow T_i^{-1}(L)$ accepted by $\mathbf{0}$-pebble acceptor

**Proof**

- Let $B$ be a nondeterministic top-down tree automaton which accepts $\overline{L}$
- We construct a $\mathbf{0}$-pebble acceptor $A$ for $T_i^{-1}(\overline{L})$, i.e., an automaton which on input $t$ decides whether $T(t)$ is accepted by $B$:
  - Simulate $T$ on $t$ and $B$
  - Simulate at the same time the behaviour of $B$ on the (virtual) output tree - this is possible as the output tree is produced top-down and can be instantly consumed by $B$
  - The simulation involves branching, whenever $T$ branches

# Step (iii)

**Lemma**

$0$-pebble acceptors only accept regular tree languages

**Proof idea**

Show that the language of $0$-pebble acceptors can be expressed by an MSO-formula:

1. Reduce $0$-pebble automaton acceptance to AGAP (Alternating Graph Accessibility)
2. Show that AGAP can be expressed in MSO

**Accessible Nodes**

Let $G = (V,E)$, $V = V_\wedge \cup V_\vee$. A node $w$ is **accesible** if

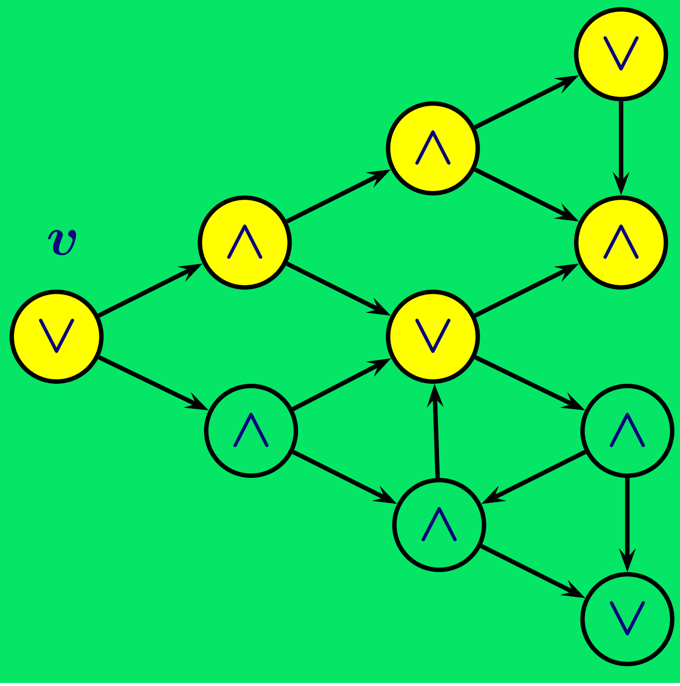
- $w \in V_\wedge$ and all successors of $w$ are accessible, or
- $w \in V_\vee$ and at least one successor of $w$ is accessible

**Definition: AGAP**

**Given:** Graph $G = (V,E)$, $V = V_\wedge \cup V_\vee$, and $v \in V$

**Question:** Is $v$ accessible?

**Example**



# 0-Pebble Acceptors, AGAP and MSO

## Construction of $G_{A,t}$

- Nodes in $V_\vee$ are the **configurations** of $A$ on $t$:
  pairs $[q,v]$, state $q$, node $v$ of $t$
- Nodes in $V_\wedge$ are $\epsilon$ and certain pairs $(\gamma_1,\gamma_2)$ of configurations
- Edges:
  - $(\gamma_1,\gamma_2) \longrightarrow \gamma_1$, $(\gamma_1,\gamma_2) \longrightarrow \gamma_2$
  - $\gamma \longrightarrow \gamma'$, if this is a step of $A$
  - $\gamma \longrightarrow \epsilon$, if $A$ can get into the accept state from $\gamma$
  - $\gamma \longrightarrow (\gamma_1,\gamma_2)$ if this is a branching step of $A$

## Facts

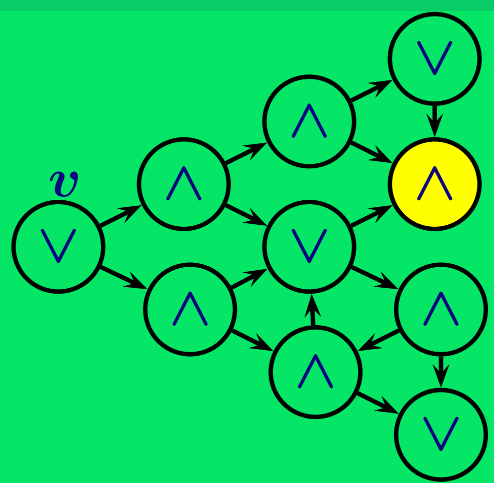
- $A$ accepts $t \iff$
  $[q,\text{root}]$ is accessible in $G_{A,t}$ with $q \in F$
- $|G_{A,t}| = O(|t|)$

## Definition: Reverse-closed Sets of Nodes

A set $S$ of nodes is **reverse-closed** $:\iff$

- if $v$ is in $V_\wedge$ and $w \in S$, for all nodes $w$ with $(v,w) \in E$, then $v \in S$
- if $v$ is in $V_\vee$ and $w \in S$, for some node $w$ with $(v,w) \in E$, then $v \in S$

## Example




## Fact

Node $v$ is accessible iff it is in every reverse-closed set of nodes

## Reverse-closed as MSO-Formula

$\forall S\ (\text{rc}(S) \longrightarrow S(v))$, where $\text{rc}(S)$ is

$$\forall x\,([V_\wedge(x) \wedge \forall y\,(E(x,y) \longrightarrow S(y))] \longrightarrow S(x)) \wedge$$
$$([V_\vee(x) \wedge \exists y\,(E(x,y) \wedge S(y))] \longrightarrow S(x))$$

# XSLT Typechecking: Summary

## Summary of proof

- Given $d_1$, $d_2$ and $T$, we can proceed as follows:
  (1) Construct the $k$-pebble acceptor $A$ for $T^{-1}(\overline{L(d_2)})$
  (2) Transform $A$ into an equivalent MSO formula $\Phi$
  (3) $\Phi$ holds for all trees $t$ for which $T(t) \notin L(d_2)$
  (4) Construct a nondeterministic bottom-up automaton $A'$ equivalent to $\neg\Phi$
  (5) Check that $L(d_1) \subseteq L(A')$
- Complexity: non-elementary

## Related work

- TypeChecking is decidable for compositions of macro tree transducers [Engelfriet, Maneth 03]
- If transformations are allowed to compare data values in the input document, type checking becomes undecidable very quickly, even for restricted types and transformations [Alon et al. 01]
- Typechecking for deterministic top-down tree transducers is more tractable. Complexity depends on exact representation of DTDs and restrictions on the transducers: between **PTIME** and **EXPTIME** [Martens, Neven 03]

Contents

# Navigation and Temporal Logics

**Remarks**

- MSO-logic offers a framework for dealing with schemas
- Close connection to automata
- Helpful in proofs
- Extends to (node-selecting) queries
- But: for many querying tasks MSO-power is not needed
- `XPath`: Navigation along paths

$\longrightarrow$ Similarity to temporal logics

- Presentation follows [Libkin 05]

# Navigation and Temporal Logics (cont.)

**Definition: An LTL-like logic: $TL^{tree}$**

- $A$, $\varphi \vee \psi$, $\neg\varphi$
- $X_*\varphi$, $X_*^-\varphi$, $\varphi U_* \psi$, $\varphi U_*^- \psi$

($* = \longrightarrow$ or $\downarrow$)

**Theorem [Marx 04]**

A unary or binary query over unordered trees is FO-definable (with $\downarrow^*$ , $\longrightarrow^*$) iff it is definable in $TL^{tree}$

**Proof idea**

Proof similar as equivalence of LTL and FO on orders [Kamp 68]

**Definition: A CTL*-like logic: $CTL^*_{past}$**

- Node formulas:
  - $A$, $\alpha \vee \alpha'$, $\neg\alpha$
  - $E\beta_*$
- Path formulas:
  - $\alpha$, $\neg\beta_*$, $\beta \vee \beta'$
  - $X_*\beta$, $X_*^-\beta$, $\beta U_* \beta'$, $\beta U_*^- \beta'$

($* = \longrightarrow$ or $\downarrow$)

**Theorem [Barcelo, Libkin 05]**

A unary or binary query over unordered trees is FO-definable (with $\downarrow^*$ , $\longrightarrow^*$) iff it is definable in $CTL^*_{past}$

**Proof idea**

Proof similar as equivalence of $CTL^*$ and FO on binary trees [Hafer, Thomas 87]

# A Related Result

**Theorem [Marx 04]**

Containment of Navigational `XPath` queries in the presence of DTDs is is in **EXPTIME**

**Proof idea**

- Navigational `XPath` can be translated into propositional dynamic logic (PDL) (over structures with $\downarrow$ , $\longrightarrow$)
- DTDs can also be expressed by PDL-formulas
- Implication for PDL is **EXPTIME**-complete

**Remark**

For much weaker fragments, containment is already **EXPTIME**-hard

## Contents

# XPath and Fragments of First-order Logic

## Characterizations of XPath

- Navigational XPath (without not and and) corresponds to positive existential first-order logic
- Different XPath axes correspond to different signatures

[Benedikt, Fan, Kuper 03]

### Proof idea

- Basic idea:
  For each node $u$ of the query tree: guess a node $h(u)$ in the document tree and check that $h$ is a "homomorphism"
- Main difficulty in proof:
  Deal with conjunctions of conditions

## Further Results on

- closure properties
- axiomatizations of equivalence

## Theorem [Marx, de Rijke 05]

Node-selecting navigational XPath queries correspond to unary two-variable first-order formulas over $\downarrow, \downarrow^*, \longrightarrow, \longrightarrow^*$

### Proof idea

Proof follows the same lines as characterization of unary temporal logic by two-variable logic

[Etessami, Vardi, Wilke]

# XPath and Full First-Order Logic

**Question**

What is needed to capture full first-order logic?

**Conditional XPath**

- **Conditional axes**: Expressions of the kind $P^+$, where $P$ is a location step
- **Conditional XPath**: Navigational XPath plus conditional axes

**Example**

$(\texttt{child} :: a[\texttt{desc} :: b \text{ or } \texttt{child} :: c])^+$

holds between $u$ and $v$ if

- $v$ is a descendant of $u$ and
- all intermediate nodes
  - are labelled with $a$ and
  - have a $c$-child or a $b$-descendant

**Theorem [Marx 04,05]**

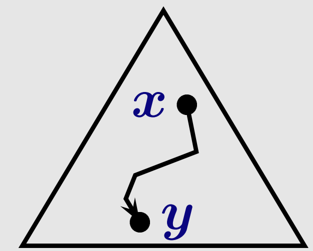
Conditional XPath corresponds exactly to first-order logic over $\downarrow^*$, $\longrightarrow^*$ (wrt node-selecting and binary queries)

# First-Order Logic Plus Regular Expressions

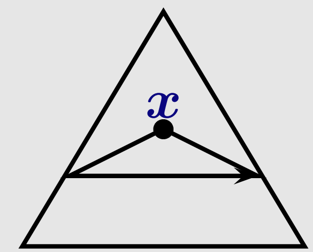
**FOREG**

First-Order Logic

+ vertical regular expressions (over paths)

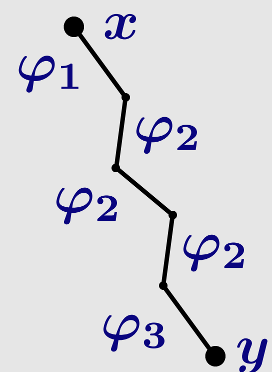



+ horizontal regular expressions (over children)




+ Nesting of formulas and regular expressions

$$[\varphi_1(s,t) \cdot \varphi_2(s,t)^* \cdot \varphi_3(s,t)]^{\downarrow}_{s,t}(x,y)$$




**Fact**

FO $\subsetneq$ FOREG $\subsetneq$ MSO

# First-Order Logic and Automata

- MSO-logic $\equiv$ Parallel tree automata
- FO-logic $\equiv$ ???

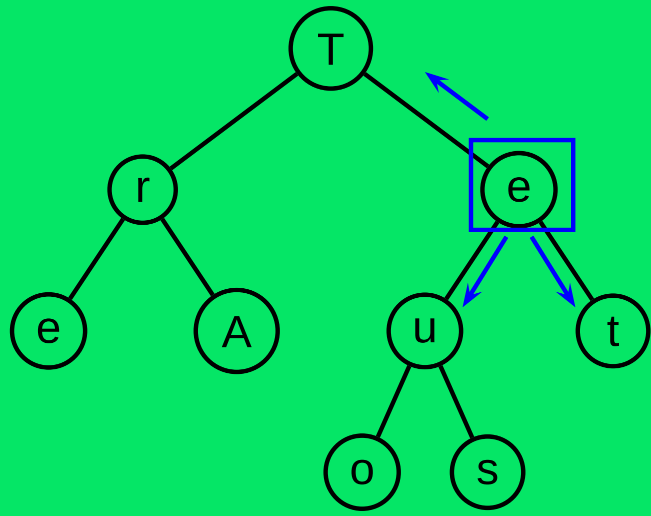
**Definition: Tree-walk automaton**

Depending on

- current state
- symbol of current node
- position of current node wrt its siblings

the automaton moves to a neighbor and takes a new state

**Illustration**




**Fact [Neven, Sch. 00]**

FO-sentences over binary trees with $\downarrow$ and $\longrightarrow$ can be evaluated by deterministic Tree-Walk automata

**Proof idea**

- Simple application of Gaifman's Theorem
- Does not hold for unranked trees

**Remark**

To capture the expressive power of Tree-walk automata one needs a bit more...

**Theorem [Neven, Sch. 00]**

- Nondet. TWA $\equiv \boldsymbol{TC^1}[\boldsymbol{FO}[\downarrow, \text{mod}]]$
- Det. TWA $\equiv \boldsymbol{DTC^1}[\boldsymbol{FO}[\downarrow, \text{mod}]]$

over binary trees with $\downarrow$ and $\longrightarrow$

## Contents

# MSO-logic vs. Unary Transitive Closure Logic

## Remarks

- On strings: MSO $\equiv$ FO $+$ unary TC
- On (binary) trees???

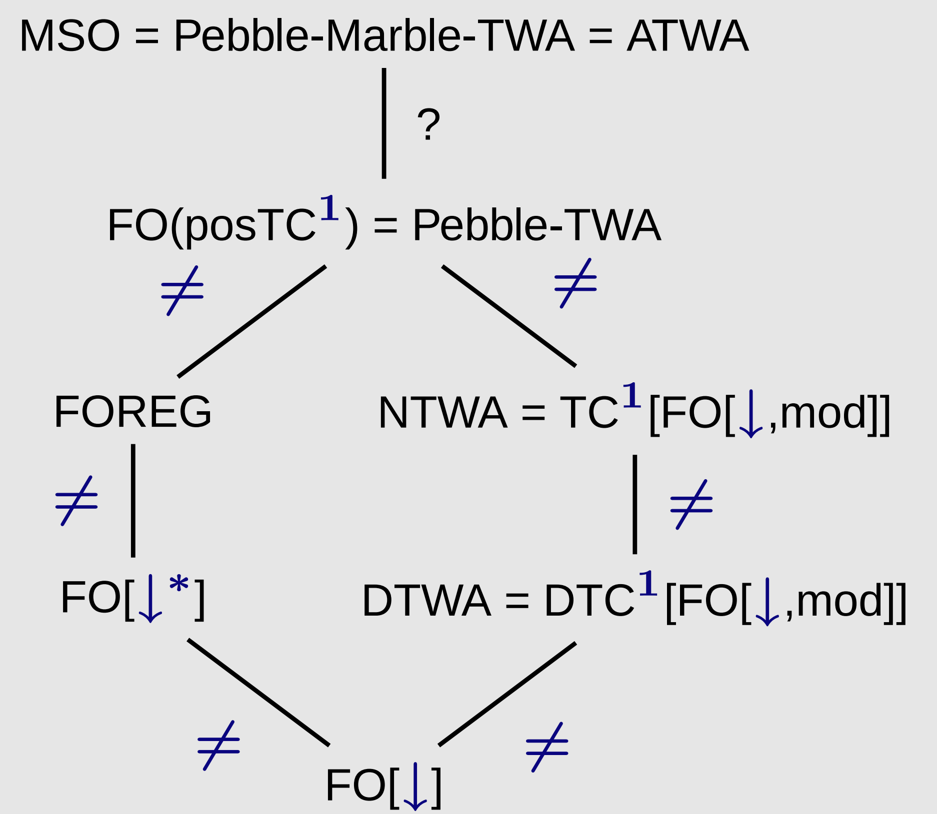
## Theorem [Engelfriet, Hoogeboom 05]

Nondeterministic pebble automata correspond to FO plus positive unary TC on binary trees

## Corollary

Positive, unary TC-logic is weaker than MSO on binary trees iff pebble automata are weaker than parallel automata

## Inclusion structure



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# Extensions

## So far...

- We have seen logics for
  - Validation, Typing
  - Navigation
  - Transformation
- What about more general queries?
  - results of higher arity?
  - joins, i.e., comparisons of data values
  - counting

## Counting

- Automata can be equipped with counting facilities, e.g.:
  Presburger tree automata: $\delta(\sigma, q)$ is Boolean combination of
  - regular expressions and
  - quantifier-free Presburger formulas like "number of children in state $q_1 =$ number of children in state $q_2$"
- Nondet. Presburger automata:
  - $\equiv$ EMSO logic
  - Whether automaton accepts all trees is undecidable
- Det. Presburger automata:
  - $\equiv$ Presburger $\mu$-formulas
  - Membership test: $O(|\mathcal{A}||t|)$
  - Non-emptiness: **PSPACE**
  - Containment: **PSPACE**

[Seidl, Sch., Muscholl, Habermehl 2004]

## Contents

# Data Values

## Remark

- In (database) queries or constraints comparisons of data values are very common
- Most XML theory concentrates on structure of trees instead

## Example queries/constraints

- Did we reserve a room for every participant?
- $\forall x$ Partic.Name$(x) \longrightarrow$ $\exists y$Room.Name$(y) \wedge x$.data $= y$.data

---

- Does every participant give at most one talk?
- $\forall x \forall y\ [x \neq y \wedge$ Talk.Speaker$(x) \wedge$ Talk.Speaker$(y)] \longrightarrow x$.data $\neq y$.data

## The setting

- We concentrate on data strings
- Access to data values only via equality tests

## Example: data string

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 3 | 3 | 2 | 2 | 7 | 17 | 17 | 3 | 4 | 5 | 2 | 3 | 3 | 4 | 4 |
| $c$ | $b$ | $c$ | $a$ | $a$ | $b$ | $b$ | $b$ | $c$ | $a$ | $b$ | $a$ | $c$ | $b$ | $b$ | $a$ | $a$ |

## Definition

- **Data string**: Finite sequence over $\Sigma \times \mathcal{D}$, where
  - $\Sigma$ finite
  - $\mathcal{D}$ infinite
- Logical language:
  - $\boxed{a(x)}$: Letter position $x$ is $a \in \Sigma$
  - order relation $<$, successor relation $+1$
  - $\boxed{\sim}$: $x \sim y$ if positions $x$ and $y$ have the same $\mathcal{D}$-value

$\longrightarrow$ Equivalence relation

# Some Known Results about $\mathbf{FO^2}$

| Over arbitray relational structures | Over strings |
|---|---|
| • Finite model property [Mortimer 75]<br>• Satisfiability **NEXPTIME**-complete [Grädel et al. 97]<br>• On structures with 1 or 2 equivalence relations: decidable [Kieroński, Otto 05]<br>• On structures with 3 equivalence relations: undecidable [Kieroński, Otto 05]<br>• On structures with a linear order: in **coNEXPTIME** [Otto]<br>• On structures with several well-orderings: undecidable [Otto] | • Satisfiability **NEXPTIME**-complete<br>• Expressive power:<br>– unary LTL and $\mathbf{\Sigma^2} \cap \mathbf{\Pi^2}$ [Etessami, Vardi, Wilke 97]<br>– variety DA [Thérien, Wilke 98]<br>– two way, partially-ordered DFA [Sch., Thérien, Vollmer 01] |

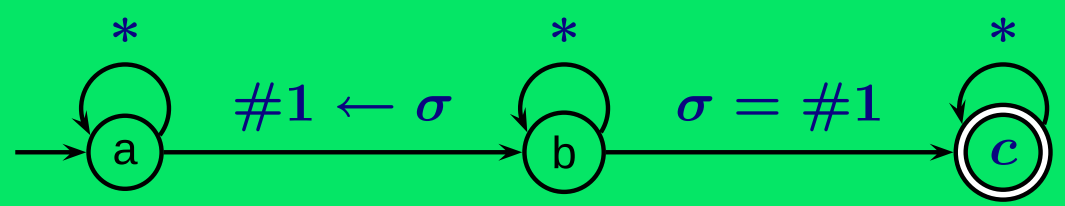
# Automata on Data Strings

## Models

- **Register-Automata**: finitely many registers for data values
- **Pebble-Automata**: finitely many pebbles with stack discipline
- Variations:

  1-way or 2-way, Deterministic or Nondeterministic

## Example



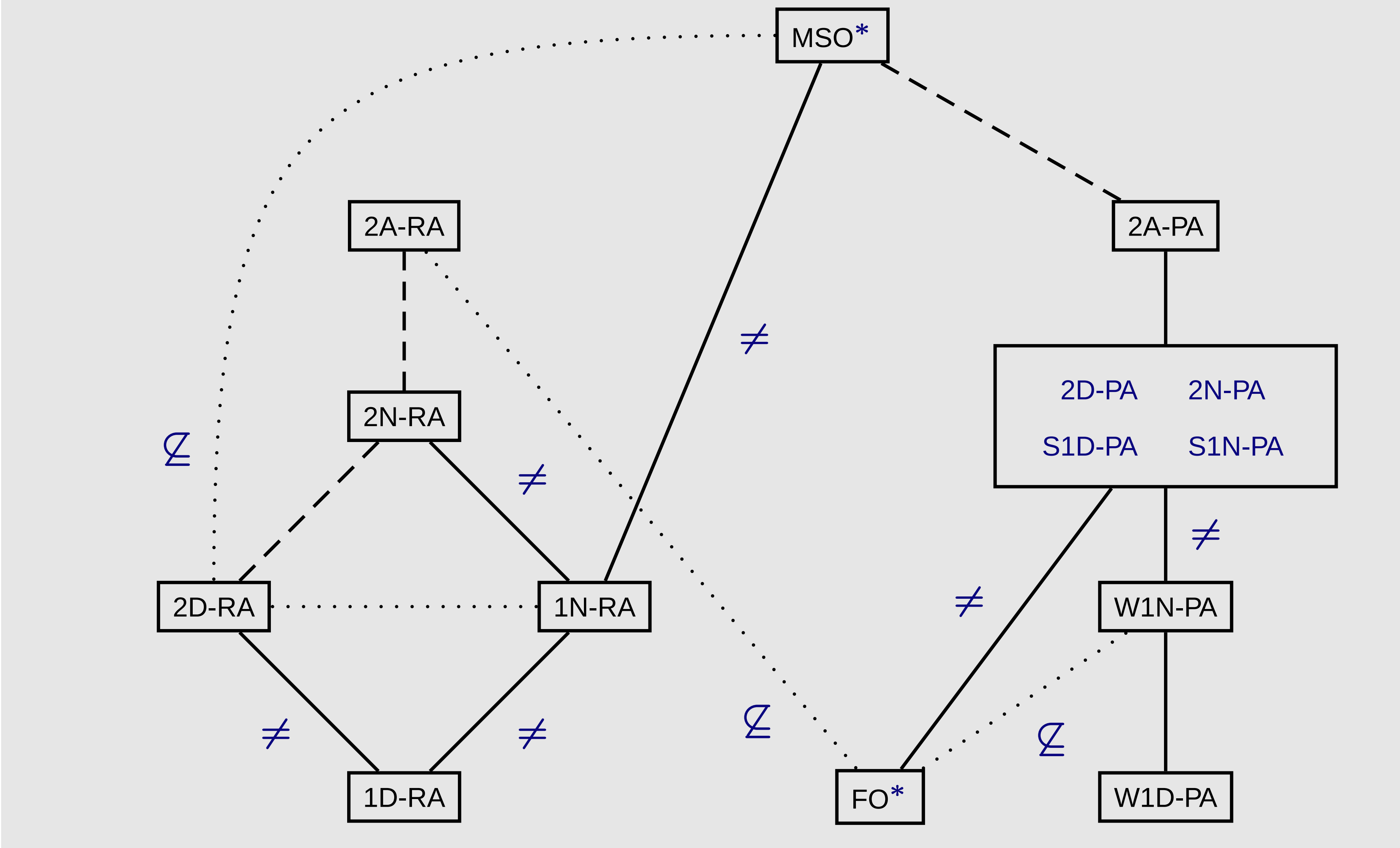

## Facts

- Non-emptiness undecidable for most models
- Exception: 1N-register automata
- This holds also for **FO** logic

$\longrightarrow$ [Kaminski, Francez 94; Neven, Sch., Vianu 01]

# Overview of Automata Models

## Inclusion Structure



# Decidability Results

## Theorem

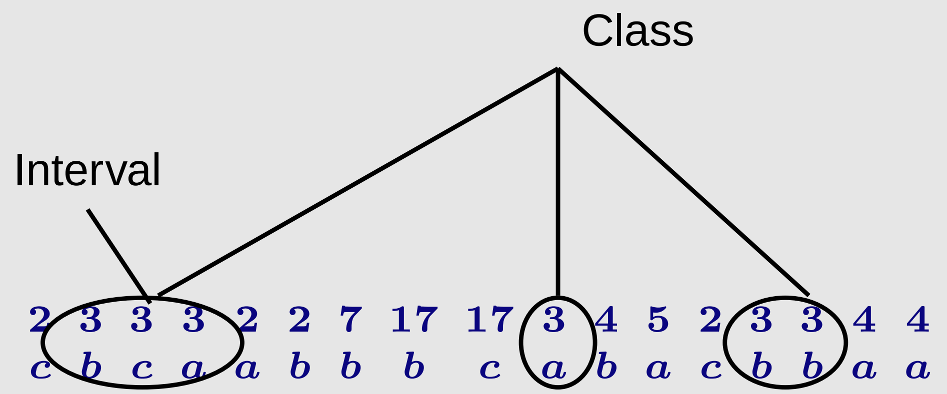
Satisfiability of **FO**$^2(+1, <)$ on data strings is decidable

## Further results

- Emptiness of multicounter automata can be reduced to satisfiability of **FO**$^2(+1, <)$ on data strings
- **FO**$^2(<)$ on data strings is complete for **NEXPTIME**
- **FO**$^2(+1)$ on data strings is hard for **NEXPTIME** (and in $2$**NEXPTIME**)
- **FO**$^3(+1)$ on data strings is undecidable

## Some Definitions

- Data string $s$:




- **Class**: all positions with the same data value
- **Interval**: (maximal set of) contiguous positions of a class
- **String projection $P(s)$**:

$$cbcaabbbcabacbbaa$$

# What $\mathrm{FO}^2$ Can Express

## Example properties

- Let $\alpha$ and $\beta$ denote unary types
- $\mathrm{FO}^2$ can express
  - **data-blind** properties, i.e., properties not using $\sim$
  - All occurrences of a type $\alpha$ are in the same class:
    $$\theta = \forall x \forall y \left( (\alpha(x) \wedge \alpha(y)) \rightarrow x \sim y \right)$$
  - Each class contains at most one occurrence of $\alpha$:
    $$\theta = \forall x \forall y \left( (\alpha(x) \wedge \alpha(y) \wedge x \sim y) \rightarrow x = y \right)$$
  - In each class, every $\alpha$ occurs before every $\beta$:
    $$\theta = \forall x \forall y \left( (\alpha(x) \wedge \beta(y) \wedge x \sim y) \rightarrow x < y \right)$$
  - Each class with an $\alpha$ also has a $\beta$:
    $$\theta = \forall x \exists y \left( \alpha(x) \rightarrow (\beta(y) \wedge x \sim y) \right)$$
  - It turns out: **That's basically all!**

# Proof Structure

## Main steps of the proof

- $$FO^2 \text{ formula } \varphi$$
  $$\downarrow$$
  $$\text{Scott normal form}$$
  $$\downarrow$$
  $$\text{Intermediate normal form}$$
  $$\downarrow$$
  $$\text{Data normal form } \psi$$
- Construct multicounter automaton $\mathcal{A}_\psi$ such that:
  - $\mathcal{A}_\psi$ accepts a string $w$ if and only if
  - there is a data string $s$ with
    - $s \models \varphi$
    - $P(s) = w$
- Check whether $L(\mathcal{A}_\psi) \neq \emptyset$

## Definition: Multicounter-automaton

- Nondeterministic string automaton (not: data string!)
- Finite number of counters
- Counter values $\geq 0$ (or reject)
- No intermediate test whether counter is 0
- Acceptance if finally all counters are 0

## Remarks

- Closely related to Petri nets
- Non-emptiness: decidable [Sacerdote, Tenney 77]
- Lower bound: **EXPSPACE**
- No elementary upper bound known

# Normalization

## Normal forms

- We transform into equivalent EMSO formulas
- **Scott normal form**: $\exists R_1, \ldots, R_k\ \forall x \forall y\ \chi \ \wedge\ \bigwedge_i \forall x \exists y\ \chi_i$
- **Intermediate normal form**:

$$\exists R_1 \cdots R_m \theta_1 \wedge \cdots \wedge \theta_n$$

- $\theta_i$:

(1) $\forall x \forall y \quad (\delta(x,y) \geq 2 \wedge \alpha(x) \wedge \beta(y) \wedge \boxed{\begin{array}{c} x \sim y \\ x \not\sim y \end{array}}) \rightarrow \boxed{\begin{array}{c} x < y \\ x > y \\ \mathit{ff} \end{array}}$

(2) $\forall x \exists y \quad \alpha(x) \rightarrow (\beta(y) \wedge \boxed{\begin{array}{c} x \sim y \\ x \not\sim y \end{array}} \wedge \boxed{\begin{array}{c} x+1 < y \\ x+1 = y \\ x = y \\ x = y+1 \\ x > y+1 \end{array}})$

- Note: $\forall\forall$ without $+1$

# Normalization (cont.)

## Normal forms (cont.)

- **Data normal form**: Disjunction of formulas

$$\exists R_1 \cdots R_n R_\# \; \theta_1 \wedge \cdots \wedge \theta_n$$

- $\theta_i$:
  - (a) data-blind
  - (b) All $\alpha$ are in the same class
  - (c) Each class contains at most one $\alpha$
  - (d) In each class, every $\alpha$ occurs before every $\beta$
  - (e) Each class with an $\alpha$ also has a $\beta$
  - (f) $R_\#$ marks the first position of each interval: $\forall x R_\#(x) \leftrightarrow \forall y(x = y + 1 \rightarrow x \not\sim y)$

## Normalization steps

$FO^2$ $\longrightarrow$ **Scott normal form:** standard

**Scott normal form**

$\longrightarrow$ **intermediate normal form:**

relatively straightforward

**Intermediate normal form**

$\longrightarrow$ **data normal form:**

- For each type $\alpha$ we capture the two left-most classes with $\alpha$ and the two rightmost classes with $\alpha$ by unary relations $R_1^\alpha, \ldots, R_4^\alpha$
- Case distinction on possible formulas (1) and (2)

$\longrightarrow$ in each case $\theta_i$ can be replaced by some “data normal” formulas

# Construction of Multicounter automaton

**Proof (cont.)**

- Recall ingredients of data normal form:
  - (a) data-blind
  - (b) All $\alpha$ are in the same class
  - (c) Each class contains at most one $\alpha$
  - (d) In each class, every $\alpha$ occurs before every $\beta$
  - (e) Each class with an $\alpha$ also has a $\beta$
  - (f) $R_{\#}$ marks the first position of each interval:

$$\forall x R_{\#}(x) \leftrightarrow \forall y(x = y + 1 \rightarrow x \not\sim y)$$

- (a), (f): straightforward
- (c), (d), (e) induce regular conditions for each class: $L$
- (b) specifies regular conditions for some special classes: $L_1, \ldots, L_k$
- Multicounter automaton $\mathcal{A}$ accepts basically shuffle of $L, L_1, \ldots, L_k$

# Construction of Multicounter automaton (cont.)

**Proof (cont.)**

- $\mathcal{A}$ accepts string projections of models of disjunctions of formulas
$$\exists R_1 \cdots R_n R_\# \; \theta_1 \wedge \cdots \wedge \theta_n$$
- $\mathcal{A}$ guesses a disjunct
- $\mathcal{A}$ guesses, for each position $R_1, \ldots, R_n, R_\#$

$\longrightarrow$ In particular: guesses intervals

- But: $\mathcal{A}$ does not know which intervals belong to the same class

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- Special classes ($L_1, \ldots, L_k$) can be checked directly (if $\alpha$ occurs in only one class this class is $R_1^\alpha$)
- To check that all other class strings are in $L$:
  - Let $\mathcal{B}$ be a string automaton for $L$
  - $\mathcal{A}$ has one counter per state of $\mathcal{B}$
  - Counter $C_q$ counts how many (non-special) class strings seen so far led to a state $q$
  - Complication: At interval border $\mathcal{A}$ can proceed from a state $q$ that was just “reached” only if $C_q \geq 2$

Contents

# Conclusion

## What we have seen

Logic is useful for the theory of XML languages:

- MSO offers framework for schema languages
- MSO $\equiv$ regular node selecting queries
- Two-variable logic $\equiv$ `XPath`
- FO-logic $\equiv$ natural extension of `XPath`
- MSO helpful in the context of transformations
- Two-variable logic with data is decidable

## Open

There remains a lot to be done, e.g.

- `XQuery`
- Automata in the presence of data values
- Practical relevance of logic-automata approach?

## Finally

Thank You!