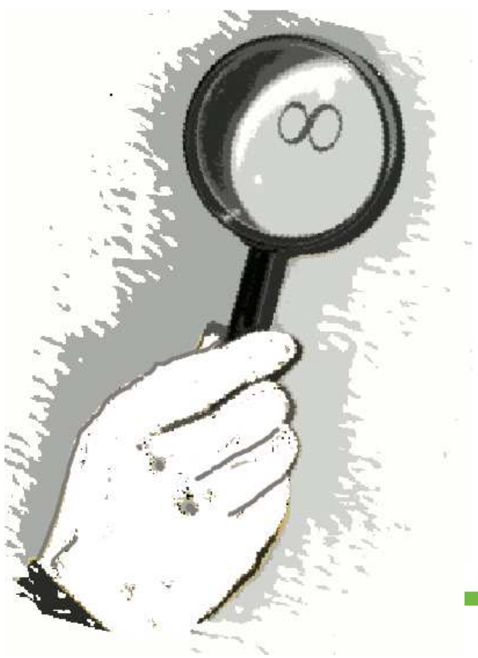
A little bit infinite?

Adding data to finitely labelled structures



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July 2008

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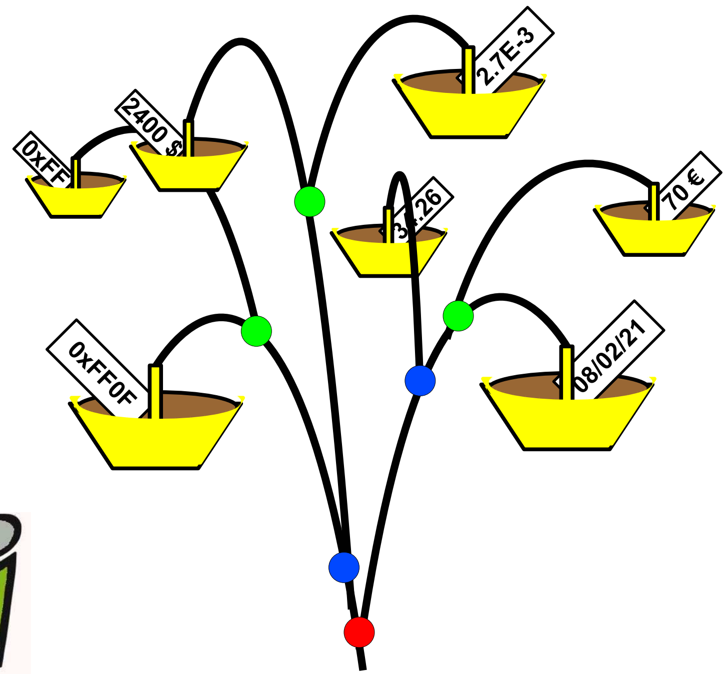

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Relational Databases

Composers from Southwest

Composers		
Name	Birth	Death
Ravel	Ciboure	Paris
Tournemire	Bordeaux	Arcachon

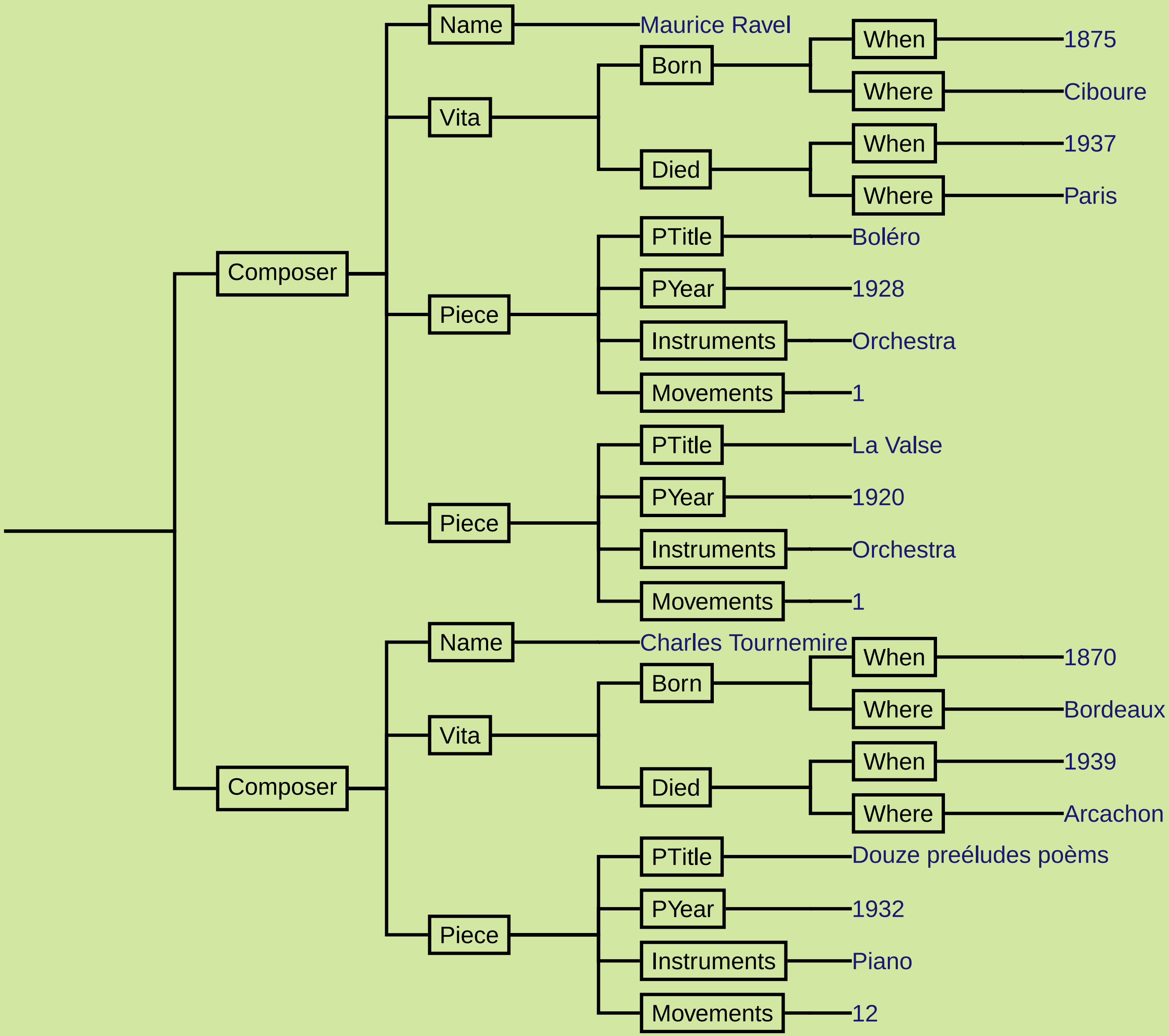
Pieces				
Name	Comp	Year	Instr	Movem
Boléro	Ravel	1928	Orch.	1
Douze Préludes	Tournemire	1932	Piano	12
La Valse	Ravel	1920	Orch.	1

```
SELECT   B.Name, B.Comp
FROM     Composers A, Pieces B
WHERE    A.Name = B.Comp AND
                   A.Birth = "Bordeaux"
```

- **Relational data:** **flat structure** & **data**
- Queries rely on **structure** and **equality of data items**:
  $$Q(x_1, x_2) \equiv \exists x_3, \ldots, x_5, y_1 \ldots, y_3 \quad \text{Pieces}(x_1, x_2, x_3, x_4, x_5) \wedge \text{Composers}(y_1, y_2, y_3) \wedge y_1 = x_2 \wedge y_3 = \text{“Bordeaux”}$$
- Integrity Constraints rely on **structure** and **equality of data items**:
  $$\forall x_1, \ldots, x_5, y_1, \ldots, y_5 \; (x_1 = y_1 \wedge x_2 = y_2) \rightarrow (x_3 = y_3 \wedge x_4 = y_5 \wedge x_5 = y_5)$$

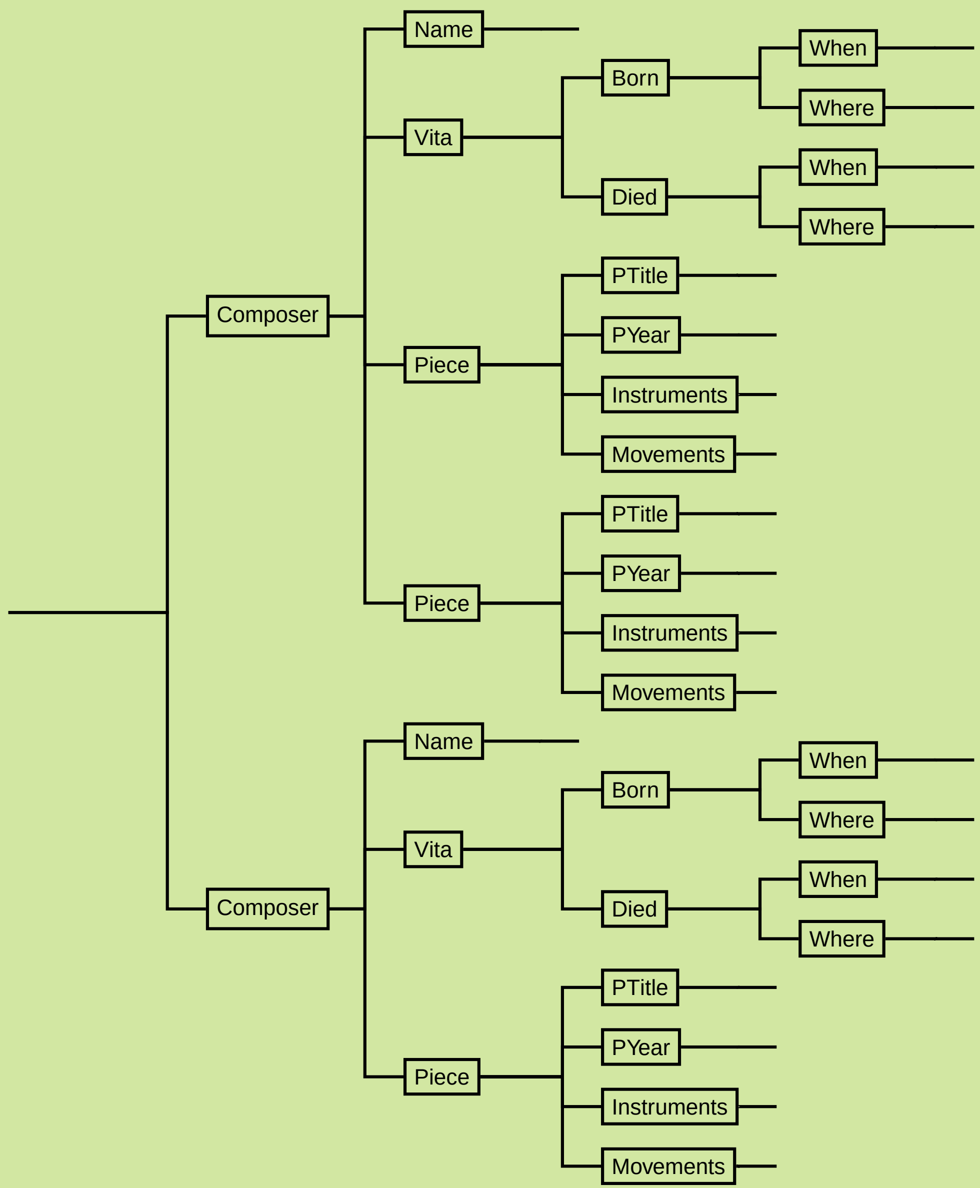
XML (1/4)

Example Tree



XML (2/4)

Example



- **XML: hierarchical structure** & **data**
- **Data model:** an XML document can be viewed as an unranked tree in which
  - inner nodes correspond to **elements**
  - leaves correspond to **data** (attributes, text content)
- For many investigations,
  - the set of tags is restricted
  - data values can be ignored

→ **Abstraction:** labeled trees over a **finite** alphabet

- Works well for foundational studies on many aspects of
  - Validation
  - Navigation
  - Transformation

→ **Foundational research on XML has largely ignored data but concentrated on finitely labeled trees**

XML (3/4)

- **There is a need for data-aware foundational XML research:**
  - **Schemas:**
    - Schemas for XML describe the allowed **structure of documents** and can specify **constraints on the data**
    - **Structure constraints** can be captured by regular tree languages (automata & logics available)
    - **Data constraints** include uniqueness, keys, foreign keys
  - **XPath:**
    - The core of XPath allows to specify navigational queries (automata & logics available)
    - But: it also allows comparisons between data
  - **Other data-aware processing tasks:**
    - Querying: XQuery
    - Transformations: XSLT
    - Data Exchange [Arenas, Libkin 05]

XML (4/4)

- An example scenario: **XML Query optimization**
  - Algorithmic problem:
    * Given XPath expressions q_1, q_2 and a schema S
    * Decide whether, for each valid document d (wrt S):
    $$q_1(d) \subseteq q_2(d)$$
  - The XPath queries might combine navigation with conditions on data values:
    * q_1: select all composers who wrote a piece in the year they died
    * q_2: select all composers whose name is unique
  - The schema S might consist of
    * structural constraints $\longrightarrow$ regular tree language L
    * and data integrity constraints
    (e.g.: each composer name occurs at most once)
  - Most of XPath navigation can be modelled by two-variable logic
  - **How to deal with data?**

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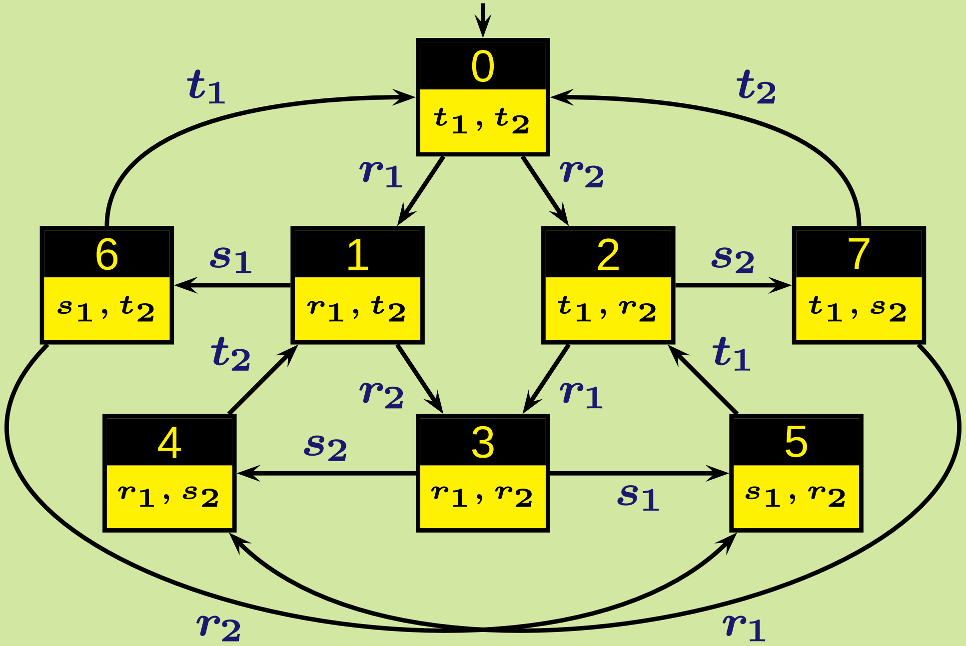
A Toy Example from Verification

A printer and two processes



- A printer and two processes
- Possible actions:
  - r_i: User i submits print **r**equest
  - s_i: Printing of request of i **s**tarts
  - t_i: Print job for user i **t**erminates

A printer and two processes (cont.)

- Example properties that might to be checked:
  - **"Local property"**: processes never request a new print job before the last one has terminated, i.e.: for each i the subrun is of the form $(r_i s_i t_i)^*$,
  - **"Global property"**: a print job must be finished before the next one is started, i.e.: between a s_i and the subsequent t_i there is no s_j or t_j, $j \neq i$

Memory Allocation

- **"Local property"**: A memory location should only be accessed after it is allocated and before it is freed

- k processes give rise to 3^k states
  ($\longrightarrow$ "state explosion")
- What if the number of processes is unknown?
- What if the number of processes changes during the computation?

The Automata Approach to Model Checking

- **Model checking:**
  - System: M
  - Property: φ
  - Does $M \models \varphi$?
- The **automata approach:**
  - Model a "real life" system as a transition system with finite state space
    → Abstract away data values, process numbers,...
  - Model executions of the system as infinite strings or trees
  - Specify properties in a logic (e.g., LTL/CTL) that allows translation into automata
  - Use decidability of non-emptiness for automata to obtain decidability of model checking

- **But sometimes the finite state space approach does not really work**
- **Sources of infinity in software systems:**
  - **Data manipulation**: integers, lists, trees, more general pointer structures
  - **Control structures**: procedures , process creation
  - **Asynchronous communication**: unbounded FIFO queues
  - **Parameters**: number of processes, duration of delays
  - **Real-time**: discrete or dense domains

[Esparza]

- There is a huge need for **Model Checking of infinite-state systems**

Current Approaches to Infinite-State Model Checking

- Infinite-State Model Checking has been an active and successful research area for many years
- **Typical approach (in a nutshell):**
  - Describe system states by some finite objects (strings, tuples of parameters)
  - Describe possible transitions from state to state
  - Device algorithms for checking reachability and/or repeated reachability
- **Examples:**
  - Timed automata [Alur, Dill 90]
  - Mutual exclusion protocols [Abdulla et al. 07]
  - Regular model checking [Bouajjani et al. 00]
- **Achievements:**
  - Model checking of linear time properties is in many cases possible
- **Still missing:**
  - Inter-state reasoning about data from infinite domains (e.g., for each i, each r_i is followed by some s_i, for an unlimited number of processes)
  - A generic framework for branching-time properties

Contents

A unifying approach

- **There are obvious similarities between the XML and the infinite-state model checking scenario:**
  - Traditional modeling uses finitely labeled structures:
    strings, trees, Kripke structures
  - There is a need to add data from infinite domains to the positions/nodes of such structures
  - It should be possible to reason about inter-node relationships between data items
- **A possible unifying approach:**
  - **Enhance finitely labeled structures by data**
  - Various possibilities:
    * One (or more) relations per node
    * A vector of data values per node
    * One data item per node
    * ...and many more

- **Parameters to choose:**
  (1) Underlying finitely labeled structures
  (2) Amount and structure of data per node
  (3) Operations and predicates on data
  (4) Expressiveness of specification language
- **Limitations:**
  - To avoid undecidability of reasoning, parameters (1) - (4) have to be chosen very carefully
- **Related work:**
  - [Autebert et al. 80]
  - [Otto 85]: Regular and context-free languages over infinite alphabets
    (Symbols have structure)
  - [Henzinger 90]: Kripke structures with one data value per word
  - [Kaminski, Francez 90]: Strings over an infinite alphabet
  - More related work will be mentioned later

Data Strings and Data Trees

- **In this talk:**
  - **We fix the structure and data parameters:**
    (1) Finite or infinite strings or trees as underlying finitely labeled structure
    (2) One data item per node/position
    (3) Only equality tests between data items
  - **We try to find (4) expressive and decidable reasoning/specification mechanisms**

Example: data string

r	r	s	r	r	t	r	s	t	s	r	t	s	t	s	t	s	t
2	5	5	3	8	5	5	2	2	8	4	8	3	3	4	4	5	5

Definition [Bouyer et al. 03]

- Data string: Finite sequence over $\Sigma \times D$, where
  - Σ finite (here: $\{r, s, t\}$)
  - D infinite (here: $\mathbb{N}$)

Regular String Languages

- Data strings extend strings
- **Regular string languages** are a very powerful concept:
  - **(1) Expressiveness:** They capture the desired languages for many kinds of applications
  - **(2) Decidability:** Automated semantic analysis possible through automata
  - **(3) Efficiency:** Model checking in linear time.
  - **(4) Closure properties:** It is hard to find a simple natural operation under which they are not (effectively) closed
  - **(5) Robustness:** Tons of characterizations

→ Regular string languages offer an ideal framework to deal with string languages:
  - Declarative specifications...
  - ..can be translated into automata...
  - ...which can be efficiently
    - evaluated,
    - manipulated and
    - analyzed semantically

- **Furthermore:** There exist canonical generalizations of regular languages for a variety of data types:
  - Infinite strings, (infinite) trees, pictures,...

→ **Obvious question:**
  - **Is there a corresponding canonical concept of "regular data languages"?**

Regular Data Languages?

- **Bad news:** There does **not** seem to be a canonical notion of regular data languages
- **Good news:** We can mimic the regular languages framework:
  - Declarative specifications...
  - ...can be translated into automata...
  - ...which can be **effectively**
    - evaluated,
    - manipulated
    - analyzed semantically
- **This talk is about the search for a good framework to deal with (string or tree) data languages:**
  - Automata for data languages
  - Logic-based specification languages
  - Their (potential) use for XML and Model Checking
  - Other approaches

Example properties of data strings

Example

$$\begin{array}{cccccccccccccccccc} r & \boxed{r} & \boxed{s} & r & r & \boxed{t} & \boxed{r} & s & t & s & r & t & s & t & s & t & \boxed{s} & \boxed{t} \\ 2 & \boxed{5} & \boxed{5} & 3 & 8 & \boxed{5} & \boxed{5} & 2 & 2 & 8 & 4 & 8 & 3 & 3 & 4 & 4 & \boxed{5} & \boxed{5} \end{array}$$

A **class** with **class string** $rstrst$

Examples

(L1) No two a-positions do have the same data value **(unary key constraint)**

(L2) There are two a-positions with the same data value

(L3) For each a-position there is a b-position with the same data value **(unary inclusion constraint)**

(L4) A print job of a user has to be printed before the next one can be requested **(“local safety”)**

(L5) Each print request of a user is eventually followed by a print **(“local liveness”)**

➞ (L1) - (L5) are **“local properties”** of the class strings

(L6) Between two successive print jobs of the same user some other user’s job has to be printed **(“global safety”)**

(L7) After each printed job a job of some other user is eventually printed **(“global liveness”)**

Contents

Register Automata (1/4)

- **A natural idea:**

  Equip finite automata with registers that can store data values

→ Register Automata

- ("Finite Memory Automata" in [Kaminski, Francez 90], but w/o labels)

Example

- Example automaton for (L6): **Between two successive print jobs of the same user some other user's job has to be printed**
- Stated differently:

  No two successive s-positions carry the same data value
- Solution: store the data value of the previous s-position in register 1 and check that it does not occur at the next s-position

$$\begin{array}{cccccccccccccccccc} r & r & s & r & r & t & r & s & t & s & r & t & s & t & s & t & s & t \\ 2 & 5 & 5 & 3 & 8 & 5 & 5 & 2 & 2 & 8 & 4 & 8 & 3 & 3 & 4 & 4 & 5 & 5 \end{array}$$

R_1	
R_2	

Register Automata (2/4)

Theorem 1 [Kaminski, Francez 90]

(a) Non-emptiness for register automata is decidable

(b) Testing $L(\mathcal{A}_1) \subseteq L(\mathcal{A}_2)$ is decidable as long as $\mathcal{A}_2$ has ≤ 2 registers

Proof idea

(a) Crux: if there is a string in $L(\mathcal{A})$, then there is one with $\leq |Q| + 1$ different data values

- There is a subtle difference between register automata models:
  (1) [Demri, Lazić 06]: data values can occur in more than register
  (2) [Kaminski, Francez 90]: they cannot
- Model (1) can simulate a Turing machine with n cells and alphabet size k with $n + k$ registers
  ➞ Non-Emptiness is **PSPACE**-complete
- If a model (2) k-register 1RA accepts any word it accepts a word of the same length with $\leq k$ data values
  ➞ Non-Emptiness is **NP**-complete

Theorem 2 [Kaminski, Francez 90]

- Universality, i.e., testing whether a register automaton accepts every data string is undecidable

Register Automata (3/4)

- Register automata can test global regular properties
  - That's simple: just ignore the data values

Theorem 3

- No register automaton can test (L4):
  "A print job of a user has to be printed before the next one can be requested"

Proof idea

- Assume some 3-register automaton $\mathcal{A}$ tests (L4)
- Consider the following input:

$$\begin{array}{ccccc} r & r & r & r & r \\ 1 & 2 & 3 & 4 & 1 \end{array}$$

R_1	4
R_2	2
R_3	3

- $\mathcal{A}$ cannot detect that process 1 has a pending print job

$\rightarrow$ Easy to generalize for arbitrary number of registers

Register Automata (4/4)

- Summary of properties of register automata:

	RegisterA
Expressiveness	
	(L2),(L6),(L7)
Decidability	
Non-emptiness	✓
Containment	–
Efficiency	
Data complexity word problem	✓
Closure properties	
Union	✓
Intersection	✓
Complement	–
Robustness	
	-

- **Variants of the basic RA model:**
  - 1-way and 2-way
  - Deterministic and non-deterministic
  - Alternating [Neven et al. 01, Demri Lazić 06]
  - Look-ahead automata [Zeitlin 06]
  - “Unification based” [Tal 99]

Contents

Pebble automata (1/3)

- A different approach: instead of registers use pebbles (pointers/heads)
- Restrict movement and placement of pebbles:
  - Pebbles are numbered $1, 2, \ldots, k$
  - Only pebble with highest number i can be moved or lifted
  - Only pebble with number $i + 1$ can be placed

➞ Pebble automata

Example

$$\begin{array}{cccccccccccccccccc} r & r & s & r & r & t & r & s & t & s & r & t & s & t & s & t & s & t \\ 2 & 5 & 5 & 3 & 8 & 5 & 5 & 2 & 2 & 8 & 4 & 8 & 3 & 3 & 4 & 4 & 5 & 5 \end{array}$$

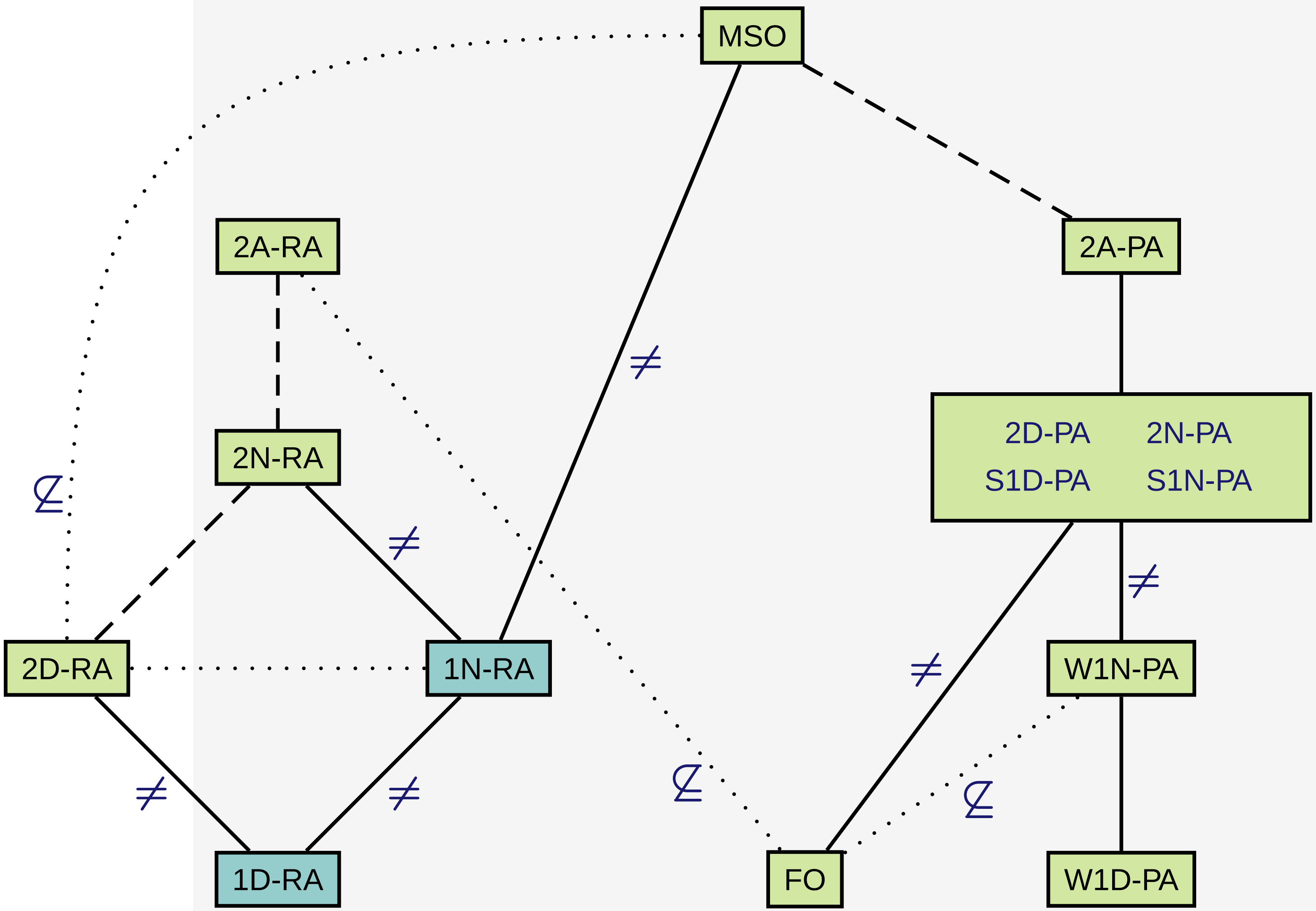
- Example automaton for (L6): **Between two successive print jobs of the same user some other user's job has to be printed**
- Again stated differently: **no two successive s-positions carry the same data value**
- **Solution:** for each s-position check that the previous s-position has a different data value

Pebble Automata (2/3)

- Pebble automata are a fairly powerful model:
  - E.g., they can express all example properties (L1) – (L7)
  - They can even express all properties that can be described by first-order logic
  - Unfortunately: first-order logic on data strings is undecidable (see below)
  - ➨ Non-emptiness of pebble automata is undecidable
- On the other hand the model is quite robust:
  - one-way and two-way, deterministic and non-deterministic pebble automata are equally expressive

	RegisterA	PebbleA
Expressiveness		
	(L2),(L6),(L7)	(L1)–(L7)
Decidability		
Non-emptiness	✓	–
Containment	–	–
Efficiency		
Data complexity word pr.	✓	✓
Closure properties		
Union	✓	✓
Intersection	✓	✓
Complement	–	✓
Robustness		
	–	✓

Pebble Automata (3/3)



(from [Neven/Sch./Vian

Contents

Class Memory Automata (1/5)

- **Intermediate state of affairs:**

  Register Automata:
  - Decidable Non-emptiness: ☺
  - Not expressive enough: ☹

  Pebble Automata:
  - Very expressive: ☺
  - Undecidable Non-emptiness; ☹

- **New approach:**
  - **Combine a global automaton with one automaton per class**
  - More precisely:
    - ∗ Transitions depend on
      - · the current input symbol (from the finite set of labels)
      - · the current state
      - · the state assumed last time in the class of the current input data value
    - ∗ The automaton accepts if
      - · the last state is in an accepting set F_g
      - · and for each class, the last state is in a set F_l

  → Class Memory Automata

  [Bojańczyk et al. 06, Björklund, Sch 07]

Class Memory Automata (2/5)

Example

- Class memory automaton for the set of data strings
  - with global pattern $(r^*sr^*t)^*$,
  - with local pattern $(rst)^*$ (for each class),
  - where at most one (singular) process prints more than once

$$\begin{array}{cccccccccccccccccc} r & r & s & r & r & t & r & s & t & s & t & r & s & t & s & t & s & t \\ 2 & 5 & 5 & 3 & 8 & 5 & 4 & 2 & 2 & 8 & 8 & 8 & 3 & 3 & 4 & 4 & 8 & 8 \end{array}$$

$$\begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline \perp & n & n & n & n & y & y & y & y & y & y & y & y & y & y & y & y & y & y \\ \hline \perp & r & r & s & r & \dot{r} & t & r & s & t & \dot{s} & \dot{t} & \dot{r} & s & t & s & t & \dot{s} & \dot{t} \\ \hline \end{array}$$

- States are of the form $\begin{array}{|c|}\hline p \\ \hline q \\ \hline\end{array}$, where
  - p remembers whether the singular process already has appeared: n, y
  - q is just the last symbol, (dotted if from the singular process)
- At the end,
  - the last state should be of the form $\begin{array}{|c|}\hline \\ \hline t \\ \hline\end{array}$ or $\begin{array}{|c|}\hline \\ \hline \dot{t} \\ \hline\end{array}$ and
  - each class should have a last state of the form $\begin{array}{|c|}\hline \\ \hline t \\ \hline\end{array}$ or $\begin{array}{|c|}\hline \\ \hline \dot{t} \\ \hline\end{array}$

Class Memory Automata (3/5)

- Class memory automata can express all properties (L1) – (L7)
- Later on we will see a precise characterization of their expressive power in terms of logic

Theorem 4

(a) Non-emptiness for class memory automata is decidable

(b) RegA $\subsetneq$ ClassMA

- The **complexity of Non-Emptiness** for class memory automata is **open**
- But there is little doubt that it is **extremely bad**:
  - Equivalent to Petri Net Reachability
  - Not even known to be primitive recursive

Proof idea for (a) [Bojańczyk et al. 06a]

- In a nutshell:
  - "Simulate" a class memory automaton $\mathcal{A}$ by a (non-data) **Multicounter Automaton**:
    - ∗ String automaton $\mathcal{A}'$ with several counters
    - ∗ $\mathcal{A}'$ has one counter C_q per state q of $\mathcal{A}$
    - ∗ C_q counts the number of classes in state q
    - ∗ Zero tests are only needed at the end of the computation:

      $C_p = 0$, for $p \notin F_l$
  - Non-emptiness for multi-counter automata is decidable

    [Mayr 81]
  - And:

$$L(\mathcal{A}) \neq \emptyset \iff L(\mathcal{A}') \neq \emptyset$$

Class Memory Automata (4/5)

Proof sketch for (b) [Björklund, S 07]

- Strictness: RAs can not express (L1)
- Isn't RegA $\subseteq$ ClassMA obvious?
- Not entirely, consider (L6): **No two successive prints by the same process**
  - The register automaton for (L6) only needs one state plus a sink state
  - How shall a ClassMA seeing $\genfrac{}{}{0pt}{}{s}{d}$ know what happened since $\genfrac{}{}{0pt}{}{s}{d}$ occurred last time?
- **Idea:** $\mathcal{A}$ "colors" positions by $\begin{array}{|c|}\hline + \\ \hline + \\ \hline\end{array}$, $\begin{array}{|c|}\hline + \\ \hline - \\ \hline\end{array}$, $\begin{array}{|c|}\hline - \\ \hline + \\ \hline\end{array}$, $\begin{array}{|c|}\hline - \\ \hline - \\ \hline\end{array}$ such that:
  - If an s-position has $\begin{array}{|c|}\hline \\ \hline + \\ \hline\end{array}$ the next s-position has $\begin{array}{|c|}\hline - \\ \hline \\ \hline\end{array}$ (and $\begin{array}{|c|}\hline \\ \hline - \\ \hline\end{array} \longrightarrow \begin{array}{|c|}\hline + \\ \hline \\ \hline\end{array}$)
  - If an s-position has $\begin{array}{|c|}\hline \\ \hline + \\ \hline\end{array}$ the next s-position **in the same class** has $\begin{array}{|c|}\hline + \\ \hline \\ \hline\end{array}$

Proof sketch for (b) (cont.)

- Of course: **if such a coloring exists, (L6) holds**: the next s-position is never the next s-position in the same class

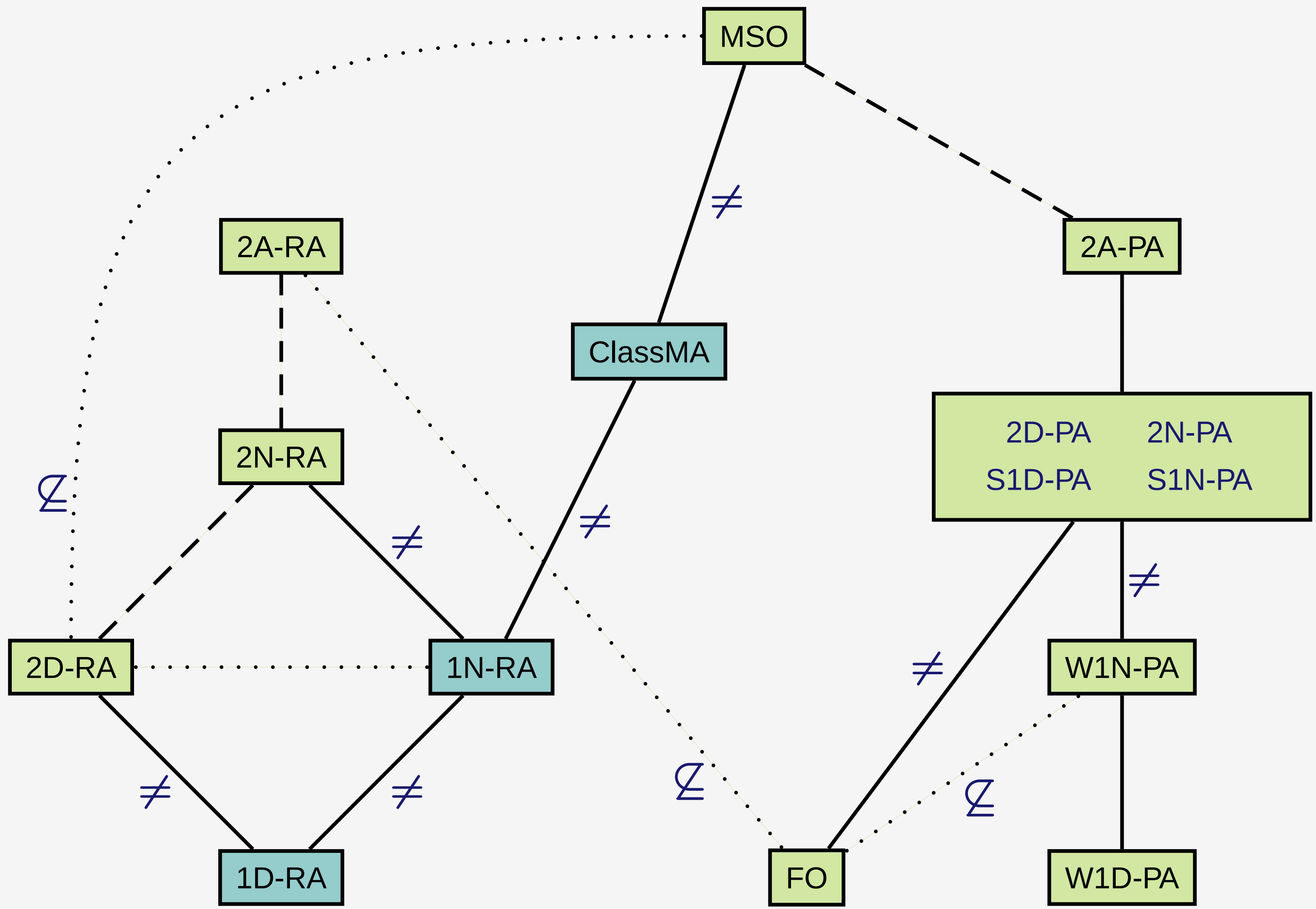
s	s	s	s	s	s	s	s	s
2	3	2	5	3	2	5	2	3
+	+	−	+	+	−	−	+	+
−	+	−	−	+	+	−	−	

- **If (L6) holds such a coloring can be constructed** by applying the following rules:
  (1) If no other rule applies: assign $\begin{array}{|c|}\hline + \\ \hline \\ \hline\end{array}$ to the rightmost s without upper color
  (2) Whenever $\begin{array}{|c|}\hline + \\ \hline \\ \hline\end{array}$ is assigned to an s-position assign $\begin{array}{|c|}\hline \\ \hline - \\ \hline\end{array}$ to its left s-neighbour and $\begin{array}{|c|}\hline \\ \hline + \\ \hline\end{array}$ to the left s-neighbour in its class
  (3) Whenever $\begin{array}{|c|}\hline \\ \hline + \\ \hline\end{array}$ is assigned to an s-position assign $\begin{array}{|c|}\hline - \\ \hline \\ \hline\end{array}$ to its right s-neighbour
- General proof of (b): similar coloring trick

Class Memory Automata (5/5)

	RegisterA	PebbleA	ClassMA	DClassMA
Expressiveness				
	(L2),(L6),(L7)	(L1)–(L7)	(L1)–(L7)	(L1)–(L5),(L7)
Decidability				
Non-emptiness	✓	–	✓	✓
Containment	–	–	–	–
Efficiency				
Data complexity word pr.	✓	✓	–	✓
Closure properties				
Union	✓	✓	✓	–
Intersection	✓	✓	✓	✓
Complement	–	✓	–	–
Robustness				
	-	✓	✓	–

Inclusion structure of Automata Models



Contents

Alternating Register Automata (1/2)

- How to turn register automata into a reasonably strong, robust and decidable model?
  - 1N-RA are pretty weak
  - 2D-RA are undecidable
- [Demri, Lazić 06]:
  - Alternating one-way register automata with **one register**: ARA_1

Theorem 5 [Demri, Lazić 06]

(a) Non-emptiness (and Containment) of ARA_1 on strings is decidable but not primitive recursive

(b) Non-emptiness of ARA_1 on ω-strings is undecidable (even with Muller acceptance)

- ARA_1 can express all properties (L1)-(L7)
- ARA_1 can not remember two data values at a time

- **Safety ARA_1** reject only in the finite (and their complement languages are closed under adding suffixes)

Theorem 6 [Lazić 06]

(a) Non-emptiness of safety ARA_1 on ω-strings is **EXPSPACE**-complete

(b) Containment of safety ARA_1 on ω-strings is decidable but not primitive recursive

Alternating Register Automata (2/2)

	RegisterA	PebbleA	ClassMA	DClassMA	ARA_1	Safe ARA_1
Expressiveness						
	(L2),(L6),(L7)	(L1)–(L7)	(L1)–(L7)	(L1)–(L5),(L7)	(L1)–(L7)	(L1),(L4),(L6)
Decidability						
Non-emptiness	✓	–	✓	✓	✓	✓
Containment	–	–	–	–	✓	✓
Efficiency						
Data complexity word pr.	✓	✓	–	✓	✓	✓
Closure properties						
Union	✓	✓	✓	–	✓	✓
Intersection	✓	✓	✓	✓	✓	✓
Complement	–	✓	–	–	✓	–
Robustness						
	-	✓	✓	–	✓	–

Contents

Logics for Data Strings/Trees

- **Automata** offer an algorithmic framework
- **Logics** offer a framework for declarative specifications
- **We will consider:**
  - Restrictions of classical first-order logic
  - Extensions of temporal logics

Logical language...			
... for strings		**... for trees**	
$a(x)$	Letter at position x is $a \in \Sigma$	$a(x)$	Label of node x is $a \in \Sigma$
$+1$	successor relation on positions	$E_{\rightarrow}$	horizontal neighbor ("next sibling")
		$E_{\downarrow}$	parent-child
$<$	order relation on positions	$E_{\Rightarrow}$	transitive closure of $E_{\rightarrow}$
		$E_{\Downarrow}$	transitive closure of $E_{\downarrow}$
$\sim$	$x \sim y$ if positions x and y have the same D-value	$\sim$	$x \sim y$ if nodes x and y have the same D-value
$\underset{\sim}{+}1$	next position in the same class		

- Of course: $\sim$ **is an equivalence relation**
- No other operations on data values, in particular no arithmetic!

A first attempt

- We know:
  - First-order logic is undecidable in general
  - First-order logic is decidable on strings
- What about First-order logic on data strings?

Theorem 7 [Bojańczyk et al. 06a]

- Satisfiability of First-Order formulas on data strings is undecidable, even for formulas with 3 variables

Proof idea

- Reduction from PCP:
  - Given: $(u_1, v_1), \ldots, (u_k, v_k)$, pairs of strings
  - Question: is there a sequence $i_1, \ldots, i_n$ such that $u_{i_1} \cdots u_{i_n} = v_{i_1} \cdots u_{i_n}$?

A bit more detail

- Encode solution candidates as data strings over $\{a, b, \#, 1, \ldots, k\}$ of the form $u\#v$
- Each occurrence of a u_i is prefixed by i: E.g., if $u_1 = aba$ and $u_2 = bb$ then 121 is encoded by $1aba2bb1aba$
- Each data value occurs exactly twice, once in u and once in v

→ corresponding positions should have the same data value

(and same number/symbol)

- Crucial: check that the sequence of data values is the same on both sides for number positions and letter positions

→ Important subformula:

$$x \sim y \rightarrow \exists z\, (x + 1 = z \wedge \exists x\, y + 1 = x \wedge z \sim x)$$

"if x and y are equivalent then their right neighbors are also equivalent"

Two Variables on Data Strings: A Useful Restriction?

- **A classical approach: Restriction to 2 variables**
- **Does this restriction give us anything useful?**
  (1) We do not have free choice...
  (2) lot of useful properties can be expressed with only two variables

Examples

(L1) No two a-positions do have the same data value
$$\forall x\ \forall y\ (x \sim y \wedge a(x) \wedge a(y)) \rightarrow x = y$$

(L2) There are two a-positions with the same data value
$$\exists x\ \exists y\ x \sim y \wedge a(x) \wedge a(y) \wedge x \neq y$$

(L3) For each a-position there is a b-position with the same data value
$$\forall x\ \exists y\ a(x) \rightarrow (b(y) \wedge x \sim y)$$

(L4) A print job of a user has to be printed before the next one can be requested
$$\forall x\ \forall y\ y = x \pm 1 \rightarrow [(r(x) \rightarrow s(s)) \wedge (s(x) \rightarrow t(y))]$$

(L5) Each print request of a user is eventually followed by a print
$$\forall x\ \exists y\ r(x) \rightarrow (s(y) \wedge x < y \wedge x \sim y)$$

(L6) Between two successive print jobs of the same user some other user's job has to be printed **not expressible**

(L7) After each printed job a job of some other user is eventually printed
$$\forall x\ \exists y\ r(x) \rightarrow (s(y) \wedge x < y \wedge x \not\sim y)$$

On the expressive power of $\mathbf{FO}^2$ on data strings (1/2)

Example

- φ_a:
  - $\forall x\, \forall y\, (x \sim y \land a(x) \land a(y)) \to x = y$
  - all a's are in different classes
- Similarly: φ_b
- $\psi_{a,b}$:
  - $\psi_{a,b} = \forall x\, \exists y\, \big(a(x) \to (b(y) \land x \sim y)\big)$
  - each class with an a also contains a b
- Similarly: $\psi_{b,a}$.

➨ – $\varphi = \varphi_a \land \varphi_b \land \psi_{a,b} \land \psi_{b,a}$ implies:
the numbers of a and b-labeled positions are equal

- In a similar fashion: number of a's, b's and c's are equal

➨ The string projection of an $\mathbf{FO}^2$-definable data language need not be context-free

On the expressive power of $\mathbf{FO}^2$ on data strings (2/2)

More example properties

- Let α and β denote unary quantifier-free formulas ("types")
- $\mathbf{FO}^2$ can express
  - data-blind properties, i.e., properties not using $\sim$
  - Each class contains at most one occurrence of α:
    $$\theta = \forall x \forall y \left((\alpha(x) \wedge \alpha(y) \wedge x \sim y) \rightarrow x = y\right)$$
  - In each class, every α occurs before every β:
    $$\theta = \forall x \forall y \left((\alpha(x) \wedge \beta(y) \wedge x \sim y) \rightarrow x < y\right)$$
  - Each class with an α also has a β:
    $$\theta = \forall x \exists y \left(\alpha(x) \rightarrow (\beta(y) \wedge x \sim y)\right)$$
  - If a position is in a different class than its successor it has type α:
    $$\theta = \forall x \forall y (\neg(x \sim y) \wedge x + 1 = y) \rightarrow \alpha(x)$$
  - That's basically all!

Theorem 8 [Bojańczyk et al. 06a]

Satisfiability of $\mathbf{FO}^2(\sim, <, +1, \pm 1)$ on data strings is decidable

Proof Sketch for Theorem 8 (1/2)

Scott and intermediate normal form

- We transform two-variable formulas into satisfiability equivalent formulas of **existential monadic second-order logic**
- **"Scott normal form"**: $\exists R_1, \ldots, R_k \, \forall x \, \forall y \, \chi \;\wedge\; \bigwedge_i \forall x \exists y \, \chi_i$
- **Intermediate normal form**:

$$\exists R_1 \cdots R_m \; \theta_1 \wedge \cdots \wedge \theta_n$$

- θ_i:

(1) $\forall x \forall y \quad (\delta(x,y) \geq 2 \wedge \alpha(x) \wedge \beta(y) \wedge \boxed{\begin{array}{c} x \sim y \\ x \not\sim y \end{array}}) \rightarrow \boxed{\begin{array}{c} x < y \\ x > y \\ \mathit{ff} \end{array}}$

(2) $\forall x \exists y \quad \alpha(x) \rightarrow (\beta(y) \wedge \boxed{\begin{array}{c} x \sim y \\ x \not\sim y \end{array}} \wedge \boxed{\begin{array}{c} x+1 < y \\ x+1 = y \\ x = y \\ x = y+1 \\ x > y+1 \end{array}})$

(3) $\forall x \forall y \quad \theta$ $\qquad$ (θ quantifier-free, DNF, no $\sim$)

- Both steps are straightforward

Proof Sketch for Theorem 8 (2/2)

Data normal form & Class Memory Automata

- **Data normal form**:
  - Disjunction of formulas $\exists R_1 \cdots R_n\ \theta_1 \wedge \cdots \wedge \theta_n$
  - θ_i:
    (a) data-blind
    (b) Each class contains at most one α
    (c) In each class, every α occurs before every β
    (d) Each class with an α also has a β
    (e) If x is in a different class than its successor has type α
- **Final Step:**
  - **Each θ_i can be recognized by a Class Memory Automaton**
  - Existential monadic quantification corresponds to nondeterminism in CMAs
  - CMAs are closed under union and intersection
  - ➨ Formulas in data normal form can be effectively translated into Class Memory Automata
- Decidability of $\mathbf{FO}^2(\sim,<,+1,\pm 1)$ follows from decidability of Non-emptiness for Class Memory Automata

- Corollary: ClassMA $\equiv \mathbf{EMSO}^2(\sim,<,+1,\pm 1)$

$\mathbf{FO^2}$ on Data Strings: Complexity

- Complexitywise, Satisfiability of $\mathbf{FO^2(\sim, <, +1)}$ is basically equivalent to Non-Emptiness of multicounter automata

➞ Unknown complexity

- **Restrictions:**
  - $\mathbf{FO^2(\sim, <)}$: complete for **NEXPTIME** [David 04]
  - $\mathbf{FO^2(\sim, +1)}$: in **3NEXPTIME** [Bojańczyk et al. 06b]
- **Extensions:**
  - $\mathbf{+2}$, $\mathbf{+3}$,...: same results
  - ω-strings: same results
  - Linear order on data values: undecidable

Two-Variable Logic on Data Trees

Theorem 9 [Bojańczyk et al. 06b]

For any **vector addition tree automaton** A, a formula $\varphi_A \in \mathbf{FO}^2(\sim, <, +1)$ can be computed such that:

$L(A) \neq \emptyset$ iff φ_A has a model

- Decidability of emptiness of vector addition tree automata is an open problem
- It is equivalent to decidability of Multiplicative Exponential Linear Logic

➞ We concentrate on $\mathbf{FO}^2(\sim, +1)$

Theorem 10 [Bojańczyk et al. 06b]

Satisfiability of $\mathbf{FO}^2(\sim, +1)$ on data trees is decidable

- The intermediate steps of the proof are similar as for data strings
- But additional techniques needed:
  - Model normalization by cut-and-paste arguments
  - ➞ Canonical “small” models that can be recognized by simpler tree automata
- **Complexity:**
  - Upper bound: **3-NEXPTIME**
  - Lower bound: **NEXPTIME**
- On trees of bounded depth: $\mathbf{FO}^2$ with all axes decidable [Björklund, Bojańczyk 07]

Consequences for XML Reasoning

- **We already know:**
  - Unary key and inclusion constraints can be expressed in $\mathbf{FO^2}(\sim, +1, <)$
- **Furthermore:**
  - Regular tree languages can be captured by $\mathbf{EMSO^2}(+1)$
  - The core of XPath without data values corresponds exactly to $\mathbf{FO^2}(+1, <)$ [Marx, de Rijke 05]
  - A simple data-aware fragment of XPath (without transitive axes) can be expressed in $\mathbf{FO^2}(\sim,+1)$

➨ **Query Containment for “simple data-aware XPath” relative to Schemas with integrity constraints is decidable**

- More results on reasoning about XML integrity constraints: [Arenas et al. 05]

Contents

Temporal Logics and the Freeze Quantifier

- $\mathbf{FO^2}$ is natural to consider from an **XML** point of view
- From a **verification** point of view it is natural to add data handling capabilities to **temporal logics**

➞ Another natural idea:
  - **"Use registers in LTL formulas"**

    [Demri, Lazić 06]

- More precisely, add the following two constructs to LTL (or another logic):
  - Unary "quantifiers" $\downarrow_i$

    (where i is a natural number)
  - Atomic formulas $\uparrow_i$
- **Informal semantics:**
  - $\downarrow_i$ stores the current data value in register i
  - $\uparrow_i$ is true if the current data value equals the value in register i

- **Syntax of LTL with Freeze:**

$$\varphi ::= \top \mid a \mid \uparrow_i \mid \varphi \wedge \varphi \mid \neg\varphi \mid \mathbf{X}\varphi \mid \mathbf{F}\varphi \mid \mathbf{G}\varphi \mid \varphi\,\mathbf{U}\,\varphi \mid \downarrow_i\varphi$$

- **Examples:**
  - (L5) each print request by a process is followed by a print for that user:

$$\mathbf{G}(r \rightarrow \downarrow_1 \mathbf{XF}(\uparrow_1 \wedge s))$$

  - (L6) Between two successive print jobs of the same user, some other user's job has to be processed:

$$\mathbf{G}\neg(r\wedge \downarrow_1 \mathbf{X}(\neg(s\wedge \uparrow_1)\,\mathbf{U}\,(s \wedge \neg \uparrow_1)))$$

LTL with Freeze

Theorem 11 [Demri, Lazić 06]

(a) Finite Satisfiability for LTL with Freeze is
  (1) undecidable in general
  (2) decidable but not primitive recursive if only 1 register is used

(b) Infinite Satisfiability for LTL with Freeze is
  - undecidable even with only 1 register

Proof idea

- More than 1 register:
  - Non-Emptiness of Minsky Counter Automata is reducible to Satisfiability of LTL with Freeze
  - ➨ Undecidability
- 1 register:
  - Satisfiability for LTL with Freeze with 1 register is basically computationally equivalent to **Non-Emptiness of Incrementing Counter Automata**:
    - ∗ Automata with counters and zero tests,
    - ∗ but: counters can always be incremented non-deterministically
  - Non-Emptiness of Incrementing Counter Automata is
    - ∗ decidable but not primitive recursive for finite strings
    - ∗ undecidable for finite strings

LTL with Freeze vs. $\mathbf{FO^2}$

- LTL with Freeze cannot express:
  - (L3) for each a-position there is a b-position with the same data value
- More generally: it cannot talk about the past
- $\mathbf{FO^2}$ cannot express:
  - (L6) Between two successive print jobs of the same user some other user's job has to be printed
- More generally: it cannot talk about "betweenness" with respect to data values

➨ LTL with Freeze and $\mathbf{FO^2}$ are incomparable

LTL with Freeze: Extensions and Restrictions

- **LTL with Freeze and past modalities:** [Demri, Lazić 06]
  - $\mathbf{X}^{-1}$, $\mathbf{G}^{-1}$, $\mathbf{F}^{-1}$, $\mathbf{U}^{-1}$
  - Can express all $\mathbf{FO^2}$ properties
  - But: Satisfiability undecidable
  - A certain fragment exactly corresponds to $\mathbf{FO^2}$
- **Safety LTL**: [Lazić 07]
  - **Safety properties:** failure is determined by a finite bad prefix
  - Safety LTL allows $\mathbf{F}$ and $\mathbf{U}$ only under an odd number of nested negations
  - **Satisfiability for Safety LTL with one register is complete for EXPSPACE**

- **Constraint LTL:** [Demri et al. 06]
  - More than 1 data value per position: "freeze variables"
  - → Undecidable
- **Constraint LTL$^\diamond$:** [Demri et al. 07]
  - Future and past modalities
  - Restricted use of data values, only two kinds of data value comparisons:
    - $x = X^k y$: variable x at current position equals variable y at current position $+k$
    - $x = \diamond y$: the current x equals some future y
  - → Finitary and Infinitary Satisfiability are decidable

Automata and Logics

	RegisterA	PebbleA	ClassMA	$\mathbf{FO^2}$	LTL & Freeze
Expressiveness					
	(L2),(L6),(L7)	(L1)–(L7)	(L1)–(L7)	(L1)–(L5),(L7)	(L1),(L2),(L4)–(L7)
Decidability					
Non-emptiness	✓	–	✓	✓	✓
Containment	–	–	–	✓	✓
Efficiency					
Data complexity word pr.	✓	✓	–	✓	✓
Closure properties					
Union	✓	✓	✓	✓	✓
Intersection	✓	✓	✓	✓	✓
Complement	–	✓	–	✓	✓
Robustness					
	-	✓	✓	?	?

Contents

Some Related Work on Data Strings

[Boyer et al. 03] Extension of register automata based on monoids

- Still can only remember a bounded number of data values
- ➨ Cannot express (L1), (L3)–(L5)

[Francez, Kaminski 03] Myhill-Nerode Theorem for data strings

[Kaminski, Tan 04] Regular expressions

- ...corresponding to unification-based register automata

[Zeitlin 06] Look-ahead register automata

- ... can guess data values
- → Closed under reversal
- Equivalent characterizations by
  - Regular expressions (stronger than the above)
  - Grammars

[Cheng, Kaminski 98] Register pushdown automata

- Decidable Non-emptiness

LTL on top of first-order logic

- [Spielmann 00]: Verification of relational transducers
- [Abdulla et al. 04]: ...even on top of MSO
- [Deutsch et al. 04]: Verification of web services
- In all cases: restricted comparison of data values of different states

Some Related Work on Data Trees

[Kaminski, Tan 06] Register automata for trees

[Jurdziński, Lazić 07]

- Alternation-free modal μ-calculus
  - Basically identical results as for LTL with Freeze
  - In particular:
    - Computationally equivalent to Incrementing Tree Counter Automata
    - Safety fragment decidable
- Alternating Automata
- XPath satisfiability

Contents

Conclusion

- **Data strings and data trees constitute a very active research area with (potential) applications in fields like Semistructured Data and Automated Verification**
- **Data strings:**
  - Attracted most attention so far
  - No obvious analogon of regular languages (so far)
  - But "logic $\longrightarrow$ automaton $\longrightarrow$ analysis" possible to some extent
  - Applicability in Verification has yet to be explored:
    * Data string approach is orthogonal to Reachability-based approaches
    * Its ability to talk about data values is limited (e.g., no arithmic)

    ➞ Is it really useful?
    * ...for other areas? (program analysis, communicating systems,...)
- **Data trees:**
  - Clearly a good model for XML data
  - Can offer a basis for data-aware static analysis
  - Needs more work
- **In both cases we need:**
  - Models with better complexity
  - Models with richer data access

Open Problems

Technical Questions:

- Precise complexity of Satisfiability of $\mathbf{FO^2(\sim, +1)}$ on data strings
- Precise complexity of Satisfiability of $\mathbf{FO^2(\sim, +1)}$ on data trees
- Is Satisfiability of $\mathbf{FO^2(\sim, <, +1)}$ on data trees decidable?
- Upper complexity bounds for Satisfiability of $\mathbf{FO^2(\sim, <, +1, \pm 1)}$ on data strings

To be explored:

- Is there a generic class of regular data (string/tree) languages?
- Find models with better complexities
- Study the trade-off between more expressive data access and complexity/decidability
- Find larger decidable fragments of data-aware XPath

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