A little bit infinite?
Adding data to finitely labelled structures

Thomas Schwentick

Grantown-on-Spey
July 2008
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- Motivation from XML
- Motivation from Verification
- Data Model
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Contents

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### Composers from Southwest

<table>
<thead>
<tr>
<th>COMPOSERS</th>
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<tbody>
<tr>
<td>Name</td>
<td>Birth</td>
<td>Death</td>
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<tr>
<td>Ravel</td>
<td>Ciboure</td>
<td>Paris</td>
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<td>Arcachon</td>
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### PIECES

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<th>Movem</th>
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<tbody>
<tr>
<td>Name</td>
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<td>Instr</td>
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SELECT B.Name, B.Comp
FROM Composers A, Pieces B
WHERE A.Name = B.Comp AND A.Birth = "Bordeaux"

- **Relational data:** flat structure & data
Relational Databases

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- Queries rely on **structure** and **equality of data items**:

\[ Q(x_1, x_2) \equiv \exists x_3, \ldots, x_5, y_1 \ldots, y_3 \]

\[ \text{Pieces}(x_1, x_2, x_3, x_4, x_5) \land \]

\[ \text{Composers}(y_1, y_2, y_3) \land \]

\[ y_1 = x_2 \land y_3 = "Bordeaux" \]
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  \[ \text{Composers}(y_1, y_2, y_3) \land \]
  \[ y_1 = x_2 \land y_3 = “Bordeaux” \]

- **Integrity Constraints** rely on **structure** and **equality of data items**:
  \[ \forall x_1, \ldots, x_5, y_1, \ldots, y_5 \]
  \[ (x_1 = y_1 \land x_2 = y_2) \rightarrow \]
  \[ (x_3 = y_3 \land x_4 = y_5 \land x_5 = y_5) \]
Example Document

⟨Composer⟩ ⟨Name⟩ Maurice Ravel ⟨/Name⟩
  ⟨Vita⟩ ⟨Born⟩ ⟨When⟩ March 3, 1875 ⟨/When⟩ ⟨Where⟩ Ciboure ⟨/Where⟩ ⟨/Born⟩
  ⟨Died⟩ ⟨When⟩ December 28, 1937 ⟨/When⟩ ⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩ ⟨/Vita⟩
  ⟨Pieces⟩
    ⟨Piece⟩ ⟨PTitle⟩ Boléro ⟨/PTitle⟩ ⟨PYear⟩ 1928 ⟨/PYear⟩
      ⟨Instrumentation⟩ Orchestra ⟨/Instrumentation⟩ ⟨Movements⟩ 1 ⟨/Movements⟩ ⟨/Piece⟩
    ⟨Piece⟩ ⟨PTitle⟩ La Valse ⟨/PTitle⟩ ⟨PYear⟩ 1920 ⟨/PYear⟩
      ⟨Instrumentation⟩ Orchestra ⟨/Instrumentation⟩ ⟨Movements⟩ 1 ⟨/Movements⟩ ⟨/Piece⟩
  ⟨Pieces⟩
  ⟨/Composer⟩

⟨Composer⟩ ⟨Name⟩ Charles Tournemire ⟨/Name⟩
  ⟨Vita⟩ ⟨Born⟩ ⟨When⟩ January 22, 1870 ⟨/When⟩ ⟨Where⟩ Bordeaux ⟨/Where⟩ ⟨/Born⟩
  ⟨Died⟩ ⟨When⟩ November 4, 1939 ⟨/When⟩ ⟨Where⟩ Arcachon ⟨/Where⟩ ⟨/Died⟩ ⟨/Vita⟩
  ⟨Pieces⟩
    ⟨Piece⟩ ⟨PTitle⟩ Douze préludes-poèmes ⟨/PTitle⟩ ⟨PYear⟩ 1932 ⟨/PYear⟩
      ⟨Instrumentation⟩ Piano ⟨/Instrumentation⟩ ⟨Movements⟩ 12 ⟨/Movements⟩ ⟨/Piece⟩
  ⟨Pieces⟩
  ⟨/Composer⟩
XML (1/4)

Example Tree

Composer

Name
Maurice Ravel
Born
When
1875
Where
Ciboure
Vita
Died
When
1937
Where
Paris
Piece
PTitle
Boléro
PYear
1928
Instruments
Orchestra
Movements
1

Composer

Name
Charles Tournemire
Born
When
1870
Where
Bordeaux
Vita
Died
When
1939
Where
Arcachon
Piece
PTitle
Douze préludes poèmes
PYear
1932
Instruments
Piano
Movements
12
Example

Composer

Name
Maurice Ravel

Vita

Born

When
1875

Where
Ciboure

Died

When
1937

Where
Paris

Piece

PTitle
Boléro

PYear
1928

Instruments
Orchestra

Movements
1

Piece

PTitle
La Valse

PYear
1920

Instruments
Orchestra

Movements
1

Composer

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Charles Tournemire

Vita

Born

When
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Where
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Piece

PTitle
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Instruments
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XML: hierarchical structure & data

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XML (2/4)

- **XML: hierarchical structure & data**
- **Data model:** an XML document can be viewed as an unranked tree in which
  - inner nodes correspond to **elements**
  - leaves correspond to **data**
    (attributes, text content)

---

**Example**

- **Composer**
  - **Name:** Maurice Ravel
  - **Vita**
    - **Born:** 1875
      - **Where:** Ciboure
    - **Died:** 1937
      - **Where:** Paris
  - **Piece**
    - **PTitle:** Boléro
      - **PYear:** 1928
    - **Instr.:** Orchestra
      - **Movements:** 1
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      - **PYear:** 1920
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      - **Movements:** 1

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  - **Name:** Charles Tournemire
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    - **Born:** 1870
      - **Where:** Bordeaux
    - **Died:** 1939
      - **Where:** Arcachon
    - **PTitle:** Douze préétudes poèmes
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XML (2/4)

**Example**

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    - **Born**: 1870
    - **Where**: Bordeaux
    - **Died**: 1939
    - **Where**: Arcachon
  - **Piece**
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    - **PYear**: 1932
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    - **Movements**: 12

**XML: hierarchical structure & data**

- **Data model**: an XML document can be viewed as an unranked tree in which
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- For many investigations,
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XML (2/4)

Example

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→ **Abstraction:**
  labeled trees over a **finite** alphabet
XML (2/4)

Example

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  - **Abstraction**: labeled trees over a **finite** alphabet
- Works well for foundational studies on many aspects of
  - Validation
  - Navigation
  - Transformation
XML: hierarchical structure & data

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- Validation
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- Transformation

Foundational research on XML has largely ignored data but concentrated on finitely labeled trees
There is a need for data-aware foundational XML research:
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> **Schemas:**
- Schemas for XML describe the allowed *structure of documents* and can specify *constraints on the data*
- **Structure constraints** can be captured by regular tree languages (automata & logics available)
- **Data constraints** include uniqueness, keys, foreign keys
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- **XPath:**
  - The core of XPath allows to specify navigational queries (automata & logics available)
  - But: it also allows comparisons between data
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  - But: it also allows comparisons between data

- **Other data-aware processing tasks:**
  - Querying: XQuery
  - Transformations: XSLT
  - Data Exchange [Arenas, Libkin 05]
An example scenario: XML Query optimization

Algorithmic problem:
An example scenario: **XML Query optimization**

- Algorithmic problem:
  - Given XPath expressions $q_1, q_2$ and a schema $S$
  - Decide whether, for each valid document $d$ (wrt $S$):
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- Most of XPath navigation can be modelled by two-variable logic

- How to deal with data?
A Toy Example from Verification

A printer and two processes

- A printer and two processes
- Possible actions:
  - $r_i$: User $i$ submits print request
  - $s_i$: Printing of request of $i$ starts
  - $t_i$: Print job for user $i$ terminates
A Toy Example from Verification

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Example properties that might to be checked:

- “Local property”: processes never request a new print job before the last one has terminated, i.e.: for each $i$ the subrun is of the form $(r_is_it_i)^*$,
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Memory Allocation

- “Local property”: A memory location should only be accessed after it is allocated and before it is freed
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- What if the number of processes changes during the computation?
The Automata Approach to Model Checking

- Model checking:
  - System: $M$
  - Property: $\varphi$
  - Does $M \models \varphi$?
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- **The automata approach:**
  - Model a "real life" system as a transition system with finite state space
    - Abstract away data values, process numbers,...
  - Model executions of the system as infinite strings or trees
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  - Use decidability of non-emptiness for automata to obtain decidability of model checking
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- **But sometimes the finite state space approach does not really work**

- **Sources of infinity in software systems:**
  - **Data manipulation**: integers, lists, trees, more general pointer structures
  - **Control structures**: procedures, process creation
  - **Asynchronous communication**: unbounded FIFO queues
  - **Parameters**: number of processes, duration of delays
  - **Real-time**: discrete or dense domains

[Esparza]
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- There is a huge need for **Model Checking of infinite-state systems**

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A little bit infinite?

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[Esparza]
• Infinite-State Model Checking has been an active and successful research area for many years

• **Typical approach (in a nutshell):**
  ▶ Describe system states by some finite objects (strings, tuples of parameters)
  ▶ Describe possible transitions from state to state
  ▶ Device algorithms for checking reachability and/or repeated reachability

• **Examples:**
  ▶ Timed automata [Alur, Dill 90]
  ▶ Mutual exclusion protocols [Abdulla et al. 07]
  ▶ Regular model checking [Bouajjani et al. 00]
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- **Achievements:**
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- **Still missing:**
  - Inter-state reasoning about data from infinite domains (e.g., for each \(i\), each \(r_i\) is followed by some \(s_i\), for an unlimited number of processes)
  - A generic framework for branching-time properties
Contents

Introduction

Data Model

Automata

Logic

Other Models

Conclusion
There are obvious similarities between the XML and the infinite-state model checking scenario:

- Traditional modeling uses finitely labeled structures:
  - strings, trees, Kripke structures

- There is a need to add data from infinite domains to the positions/nodes of such structures

- It should be possible to reason about inter-node relationships between data items
A unifying approach

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- A possible unifying approach:
  - Enhance finitely labeled structures by data
  - Various possibilities:
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    - A vector of data values per node
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A little bit infinite? Thomas Schwentick
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- Related work:
  - [Autebert et al. 80]
  - [Otto 85]: Regular and context-free languages over infinite alphabets (Symbols have structure)
  - [Henzinger 90]: Kripke structures with one data value per word
  - [Kaminski, Francez 90]: Strings over an infinite alphabet
  - More related work will be mentioned later
Data Strings and Data Trees

- In this talk:

  We fix the structure and data parameters:
  1. Finite or infinite strings or trees as underlying finitely labeled structure
  2. One data item per node/position
  3. Only equality tests between data items
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- We try to find (4) expressive and decidable reasoning/specification mechanisms

Example: data string

```
r  r  s  r  r  t  r  s  t  s  r  t  s  t  s  t
2  5  5  3  8  5  5  2  2  8  4  8  3  4  4  5  5
```

Definition [Bouyer et al. 03]

- **Data string**: Finite sequence over $\Sigma \times D$, where
  - $\Sigma$ finite (here: $\{r, s, t\}$)
  - $D$ infinite (here: $\mathbb{N}$)
Regular String Languages

- Data strings extend strings
- **Regular string languages** are a very powerful concept:
  - **Expressiveness**: They capture the desired languages for many kinds of applications
  - **Decidability**: Automated semantic analysis possible through automata
  - **Efficiency**: Model checking in linear time.
  - **Closure properties**: It is hard to find a simple natural operation under which they are not (effectively) closed
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→ Regular string languages offer an ideal framework to deal with string languages:
  - Declarative specifications...
  - ...can be translated into automata...
  - ...which can be efficiently
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    - manipulated and
    - analyzed semantically

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- **Furthermore:** There exist canonical generalizations of regular languages for a variety of data types:
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→ **Obvious question:**
  - Is there a corresponding canonical concept of "regular data languages"?
• **Bad news:** There does not seem to be a canonical notion of regular data languages
Regular Data Languages?

- **Bad news:** There does **not** seem to be a canonical notion of regular data languages

- **Good news:** We can mimic the regular languages framework:
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- **This talk is about the search for a good framework to deal with (string or tree) data languages:**
  - Automata for data languages
  - Logic-based specification languages
  - Their (potential) use for XML and Model Checking
  - Other approaches
Example properties of data strings

<table>
<thead>
<tr>
<th>Example</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>r r s r r t r s t s t s t s t</td>
</tr>
<tr>
<td></td>
<td>2 5 5 3 8 5 5 2 2 8 4 8 3 3 4 4 5 5</td>
</tr>
</tbody>
</table>
## Example properties of data strings

### Example

<table>
<thead>
<tr>
<th></th>
<th>r</th>
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<th>r</th>
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</tbody>
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A **class** with **class string** `rstrst`
Example properties of data strings

A class with class string $rstrst$

### Examples

(L1) No two $a$-positions do have the same data value

(unary key constraint)

(L2) There are two $a$-positions with the same data value

(L3) For each $a$-position there is a $b$-position with the same data value

(unary inclusion constraint)

(L4) A print job of a user has to be printed before the next one can be requested

(“local safety”)

(L5) Each print request of a user is eventually followed by a print

(“local liveness”)

→ (L1) - (L5) are “local properties” of the class strings
### Example properties of data strings

**Example**

<table>
<thead>
<tr>
<th>r</th>
<th>r</th>
<th>s</th>
<th>r</th>
<th>r</th>
<th>t</th>
<th>r</th>
<th>s</th>
<th>t</th>
<th>s</th>
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<th>t</th>
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<td>8</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

A **class** with **class string** `rstrst`

**Examples**

1. No two **a**-positions do have the same data value  
   *(unary key constraint)*
2. There are two **a**-positions with the same data value
3. For each **a**-position there is a **b**-position with the same data value  
   *(unary inclusion constraint)*
4. A print job of a user has to be printed before the next one can be requested  
   *(“local safety”)*
5. Each print request of a user is eventually followed by a print  
   *(“local liveness”)*

→ (L1) - (L5) are **“local properties”** of the class strings

6. Between two successive print jobs of the same user some other user’s job has to be printed  
   *(“global safety”)*

7. After each printed job a job of some other user is eventually printed  
   *(“global liveness”)*
Contents

- Introduction
- Data Model

**Automata**

- Register Automata
  - Pebble Automata
  - Class Memory Automata
  - Alternating Register Automata

- Logic
- Other Models
- Conclusion
A natural idea:
Equip finite automata with registers that can store data values.
• **A natural idea:** Equip finite automata with registers that can store data values

→ Register Automata
Register Automata (1/4)

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Example

Example automaton for (L6): Between two successive print jobs of the same user some other user’s job has to be printed

Stated differently:
No two successive s-positions carry the same data value

Solution: store the data value of the previous s-position in register 1 and check that it does not occur at the next s-position

\[
\begin{array}{cccccccccccccc}
  r & r & s & r & r & t & r & s & t & s & r & t & s & t & s & t & t \\
  2 & 5 & 5 & 3 & 8 & 5 & 5 & 2 & 2 & 8 & 4 & 8 & 3 & 3 & 4 & 4 & 5 & 5 \\
\end{array}
\]

\[
\begin{array}{c}
  R_1 \\
  R_2 \\
\end{array}
\]
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2.5 5 3 8 5 5 2 2 8 4 8 3 3 4 4 5 5
```

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\begin{array}{c}
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    \bot \\
    \hline
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\end{array}
\]

\[
\begin{array}{cc}
  R_1 & 5 \\
  R_2 & 1 \\
\end{array}
\]
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- A little bit infinite? Thomas Schwentick
A natural idea:
Equip finite automata with registers that can store data values

⇒ Register Automata

("Finite Memory Automata" in [Kaminski, Francez 90], but w/o labels)

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2 5 5 3 8 5 5 2 2 8 4 8 3 3 4 4 5 5
```

```
<table>
<thead>
<tr>
<th></th>
<th>R1</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>R2</td>
<td>8</td>
<td></td>
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  2 & 5 & 5 & 3 & 8 & 5 & 5 & 2 & 2 & 8 & 4 & 8 & 3 & 3 & 4 & 4 & 5 & 5 \\
\end{array}
\]

\[
\begin{array}{cccc}
  R_1 & 5 \\
  R_2 & 8 \\
\end{array}
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```
r r s r r t r s t s r t s t s t
2 5 5 3 8 5 5 2 2 8 4 8 3 3 4 4 5 5
```

```
R_1 2
R_2 8
```
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```

\[
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\end{array}
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```
  r r s r r t r s t s r t s t s t
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  R1 8
  R2 ⊥
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<tr>
<td>$R_1$</td>
<td>8</td>
</tr>
<tr>
<td>$R_2$</td>
<td>4</td>
</tr>
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</table>
Register Automata (1/4)

- **A natural idea:** Equip finite automata with registers that can store data values

  Register Automata

- (“Finite Memory Automata” in [Kaminski, Francez 90], but w/o labels)

Example

- Example automaton for (L6): **Between two successive print jobs of the same user some other user’s job has to be printed**

  Stated differently: **No two successive s-positions carry the same data value**

- Solution: store the data value of the previous s-position in register 1 and check that it does not occur at the next s-position

```
  r r s r r t r s t s t s t s t
  2 5 5 3 8 5 5 2 2 8 4 8 3 3 4 4 5 5

  R_1 3
  R_2 4
```
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```plaintext
r  r  s  r  r  t  r  s  t  s  r  t  s  t  s  t
2  5  5  3  8  5  5  2  2  8  4  8  3  3  4  4  5  5

R_1  3
R_2  4
```
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\[
\begin{array}{cccccccccccccccc}
  r & r & s & r & r & t & r & s & t & s & t & s & t & s & t & s & t \\
  2 & 5 & 5 & 3 & 8 & 5 & 5 & 2 & 2 & 8 & 4 & 8 & 3 & 3 & 4 & 4 & 5 & 5 \\
\end{array}
\]

\[
\begin{array}{ccc}
  R_1 & 4 \\
  R_2 & \bot \\
\end{array}
\]

A little bit infinite? Thomas Schwentick
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```
r r s r r t r s t s r t s t s t
2 5 5 3 8 5 5 2 2 8 4 8 3 3 4 4 5 5
```

- **Solution:**
  - **Register 1 (R1):** Value 4
  - **Register 2 (R2):** Value ⊥ (bottom)

A little bit infinite? Thomas Schwentick
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\[
\begin{array}{cccccccccccc}
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  2 & 5 & 5 & 3 & 8 & 5 & 5 & 2 & 2 & 8 & 4 & 3 & 4 & 4 & 5 & 5 \\
\end{array}
\]

\[
\begin{array}{c|c}
  R_1 & 5 \\
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\end{array}
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\[
\begin{array}{cccccccccccc}
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Theorem 1 [Kaminski, Francez 90]

(a) Non-emptiness for register automata is decidable

(b) Testing \( L(A_1) \subseteq L(A_2) \) is decidable as long as \( A_2 \) has \( \leq 2 \) registers
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- There is a subtle difference between register automata models:
  1. [Demri, Lazić 06]: data values can occur in more than register
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- Model (1) can simulate a Turing machine with $n$ cells and alphabet size $k$ with $n + k$ registers
  → Non-Emptiness is $\text{PSPACE}$-complete
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Theorem 2 [Kaminski, Francez 90]

- Universality, i.e., testing whether a register automaton accepts every data string is undecidable
• Register automata can test global regular properties
  ▶ That’s simple: just ignore the data values
Register Automata (3/4)

- Register automata can test global regular properties
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**Theorem 3**

- No register automaton can test (L4):
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Proof idea

- Assume some 3-register automaton $A$ tests (L4)
- Consider the following input:

\[
\begin{array}{cccc}
 r & r & r & r \\
 1 & 2 & 3 & 4 \\
\end{array}
\begin{array}{c}
 R_1 \\
 4 \\
 R_2 \\
 2 \\
 R_3 \\
 3 \\
\end{array}
\]
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```
  r r r r r
1 2 3 4 1
```

```
  R_1 4
  R_2 2
  R_3 3
```
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```
   r  r  r  r  r
  1  2  3  4  1

  R_1 4
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```

- $A$ cannot detect that process 1 has a pending print job
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- Assume some 3-register automaton $A$ tests (L4)
- Consider the following input:

$$
\begin{array}{cccccc}
R_1 & R_2 & R_3 & 4 & 2 & 3 \\
1 & 2 & 3 & 4 & 1 & \\
\end{array}
$$

- $A$ cannot detect that process 1 has a pending print job

$\Rightarrow$ Easy to generalize for arbitrary number of registers
### Summary of properties of register automata:

<table>
<thead>
<tr>
<th>Property</th>
<th>RegisterA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expressiveness</strong></td>
<td>(L2),(L6),(L7)</td>
</tr>
<tr>
<td><strong>Decidability</strong></td>
<td>✓</td>
</tr>
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<td>Non-emptiness</td>
<td>✓</td>
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<td>Containment</td>
<td>–</td>
</tr>
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A little bit infinite? — Thomas Schwentick
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### Variants of the basic RA model:
- 1-way and 2-way
- Deterministic and non-deterministic
- Alternating
  - [Neven et al. 01, Demri Lazić 06]
- Look-ahead automata [Zeitlin 06]
- “Unification based” [Tal 99]
Contents

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**Automata**
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  - **Pebble Automata**
    - Class Memory Automata
    - Alternating Register Automata
  - Logic
  - Other Models
  - Conclusion

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A different approach: instead of registers use pebbles (pointers/heads)

Restrict movement and placement of pebbles:
- Pebbles are numbered $1, 2, \ldots, k$
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Pebble automata

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Example

<table>
<thead>
<tr>
<th>$r$</th>
<th>$r$</th>
<th>$s$</th>
<th>$r$</th>
<th>$r$</th>
<th>$t$</th>
<th>$r$</th>
<th>$s$</th>
<th>$t$</th>
<th>$s$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>8</td>
<td>5</td>
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<td>2</td>
<td>2</td>
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<td>4</td>
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<th>s</th>
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<th>r</th>
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<th>s</th>
<th>t</th>
<th>s</th>
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\end{array}
```

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→ Pebble automata

Example

```
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  2  5  5  3  8  5  5  2  2

  s  r  t  s  t  s  t  s  t  s  t
  8  4  8  3  3  4  4  5  5
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→ Pebble automata

**Example**

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\begin{array}{cccccccccccc}
\text{r} & \text{r} & \text{s} & \text{r} & \text{r} & \text{t} & \text{r} & \text{s} & \text{t} & \text{s} & \text{t} & \text{s} \\
2 & 5 & 5 & 3 & 8 & 5 & 5 & 2 & 2 & 8 & 4 & 3 & 3 & 4 & 4 & 5 & 5 & 1
\end{array}
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| 2 | 5 | 5 | 3 | 8 | 5 | 5 | 2 | 2 | 8 | 4 | 8 | 3 | 3 | 4 | 4 | 5 | 5 | 2 | 1 |

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A little bit infinite? Thomas Schwentick
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Example automaton for (L6):
Between two successive print jobs of the same user some other user’s job has to be printed
Again stated differently: no two successive $s$-positions carry the same data value
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<tr>
<th></th>
<th>RegisterA</th>
<th>PebbleA</th>
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<tbody>
<tr>
<td><strong>Expressiveness</strong></td>
<td></td>
<td></td>
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<tr>
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<td>(L2),(L6),(L7)</td>
<td>(L1)–(L7)</td>
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<tr>
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<td></td>
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<tr>
<td>Non-emptiness</td>
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<td>–</td>
</tr>
<tr>
<td>Containment</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td><strong>Efficiency</strong></td>
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<td>✓</td>
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<tr>
<td>Union</td>
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<td>Intersection</td>
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<td>Complement</td>
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<tr>
<td><strong>Robustness</strong></td>
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</table>
Pebble Automata (3/3)

(from Neven/Sch./Vian...)

A little bit infinite? Thomas Schwentick...
Contents

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→ Class Memory Automata
   [Bojańczyk et al. 06, Björklund, Sch 07]
Class Memory Automata (2/5)

Example

- Class memory automaton for the set of data strings
  - with global pattern \((r^*sr^*t)^*\),
  - with local pattern \((rst)^*\) (for each class),
  - where at most one (singular) process prints more than once

```
  r  r  s  r  r  t  r  s  t  s  t  r  s  t  s  t
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<th>5</th>
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<th>3</th>
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⊥ n n n n y
⊥ r r s r .r
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<th>s</th>
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<th>r</th>
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<td>3</td>
<td>4</td>
<td>4</td>
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```
\[
\begin{array}{cccccccccccc}
\text{r} & \text{r} & \text{s} & \text{r} & \text{r} & \text{t} & \text{r} & \text{s} & \text{t} & \text{s} & \text{t} & \text{t} \\
2 & 5 & 5 & 3 & 8 & 5 & 4 & 2 & 8 & 8 & 3 & 4 & 4 & 8 & 8
\end{array}
\]

\[
\begin{array}{cccccccccccc}
\bot & \text{n} & \text{n} & \text{n} & \text{n} & \text{y} & \text{?} \\
\bot & \text{r} & \text{r} & \text{s} & \text{r} & \hat{r}
\end{array}
\]

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\[
\begin{array}{cccccccccccc}
  r & r & s & r & r & t & r & s & t & s & t & r & s & t & s & t \\
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\begin{array}{cccccccccccccc}
  & & r & r & s & r & r & t & r & s & t & s & t & t \\
2 & 5 & 5 & 3 & 8 & 5 & 4 & 2 & 2 & 8 & 8 & 3 & 3 & 4 & 4 & 8 & 8
\end{array}
\]

\[
\begin{array}{cccccccc}
  \downarrow & n & n & n & n & y & y & y \\
  \downarrow & r & r & s & r & t & r & s
\end{array}
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  - with local pattern \((rst)^*\) (for each class),
  - where at most one (singular) process prints more than once

```
2 5 5 3 8 5 4 2 8 8 3 4 4 8 8
n n n y y y y y y
r r s r t r s t t t
```

- States are of the form \([p,q]\), where
  - \(p\) remembers whether the singular process already has appeared: \(n, y\)
  - \(q\) is just the last symbol, (dotted if from the singular process)
Class Memory Automata (2/5)

### Example

- Class memory automaton for the set of data strings
  - with global pattern $(r^*sr^*t)^*$,
  - with local pattern $(rst)^*$ (for each class),
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<table>
<thead>
<tr>
<th>$r$</th>
<th>$r$</th>
<th>$s$</th>
<th>$r$</th>
<th>$r$</th>
<th>$t$</th>
<th>$r$</th>
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<tbody>
<tr>
<td>2</td>
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<td>5</td>
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<td>5</td>
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<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

- States are of the form $[\overline{p} \overline{q}]$, where
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  - $q$ is just the last symbol, (dotted if from the singular process)
Class Memory Automata (2/5)

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- Class memory automaton for the set of data strings
  - with global pattern \((r^*sr^*t)^*\),
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```
2 5 5 3 8 5 4 2 2 8 8 8 3 3 4 4 8 8
n n n y y y y y y y y y
r r s r r t r s t t t
```

- States are of the form \([p\ q]\), where
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  - with global pattern \((r^*sr^*t)^*\),
  - with local pattern \((rst)^*\) (for each class),
  - where at most one (singular) process prints more than once

```
  r  r  s  r  r  t  r  s  t  s  t  r  s  t  s  t
  2  5  5  3  8  5  4  2  8  8  8  3  4  4  8  8

  \(\bot\) n n n n y y y y y y y y
  \(\bot\) r r s r \(\cdot\) t r s t \(\cdot\) i \(\cdot\) r s t
```

- States are of the form \(p \overline{q}\), where
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<tbody>
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<td>r</td>
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<td>t</td>
<td>r</td>
<td>s</td>
<td>(\hat{s})</td>
<td>(\hat{t})</td>
<td>s</td>
<td>s</td>
</tr>
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Class Memory Automata (2/5)

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\[
\begin{array}{cccccccccccc}
  r & r & s & r & r & s & t & r & s & s & t & s \\
  2 & 5 & 5 & 3 & 8 & 5 & 4 & 2 & 2 & 8 & 8 & 3 \\
\end{array}
\]

\[
\begin{array}{cccccccccc}
  \underline{r} & \underline{n} & n & n & y & y & y & y & y & y \\
  \underline{r} & r & s & r & t & r & s & t & s & t \\
\end{array}
\]

- States are of the form \([p \downarrow q]\), where
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Class Memory Automata (2/5)

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\[
\begin{array}{cccccccccccc}
  r & r & s & r & r & t & r & s & t & s & t & r & s & t & s & t \\
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\end{array}
\]

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Class Memory Automata (2/5)

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\[
\begin{array}{cccccccccccc}
  r & r & s & r & r & t & r & s & t & s & t & r & s & t & s & t \\
  2 & 5 & 5 & 3 & 8 & 5 & 4 & 2 & 8 & 8 & 8 & 3 & 3 & 4 & 8 & 8 \\
\end{array}
\]

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- At the end,
  - the last state should be of the form \([t] \text{ or } [\dot{t}]\)
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\[
\begin{array}{cccccccccccc}
r & r & s & r & r & s & t & r & s & t & s & t \\
2 & 5 & 5 & 3 & 8 & 5 & 4 & 2 & 2 & 8 & 8 & 8 \\
\end{array}
\]

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  - each class should have a last state of the form \(t\) or \(\dot{t}\)
Class Memory Automata (3/5)

- Class memory automata can express all properties (L1) – (L7)
- Later on we will see a precise characterization of their expressive power in terms of logic
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Theorem 4

(a) Non-emptiness for class memory automata is decidable
(b) \( \text{RegA} \subsetneq \text{ClassMA} \)
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- The **complexity of Non-Emptiness** for class memory automata is **open**
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- The complexity of Non-Emptiness for class memory automata is open
- But there is little doubt that it is extremely bad:
  - Equivalent to Petri Net Reachability
  - Not even known to be primitive recursive
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Proof idea for (a) [Bojańczyk et al. 06a]

- In a nutshell:
  - “Simulate” a class memory automaton $\mathcal{A}$ by a (non-data) **Multicounter Automaton**:
    - String automaton $\mathcal{A}'$ with several counters
    - $\mathcal{A}'$ has one counter $C_q$ per state $q$ of $\mathcal{A}$
    - $C_q$ counts the number of classes in state $q$
  - Zero tests are only needed at the end of the computation: $C_p = 0$, for $p \notin F_l$
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### Theorem 4

<table>
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<tr>
<th>(a) Non-emptiness for class memory automata is decidable</th>
</tr>
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**Proof idea for (a) [Bojańczyk et al. 06a]**

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    - $C_q$ counts the number of classes in state $q$
    - Zero tests are only needed at the end of the computation: $C_p = 0$, for $p \notin F_t$
  - Non-emptiness for multi-counter automata is decidable [Mayr 81]
  - And:
    \[
    L(\mathcal{A}) \neq \emptyset \iff L(\mathcal{A}') \neq \emptyset
    \]
Proof sketch for (b) [Björklund, S 07]

- Strictness: RAs can not express (L1)
**Proof sketch for (b) [Björklund, S 07]**

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- Not entirely, consider (L6): No two successive prints by the same process
  - The register automaton for (L6) only needs one state plus a sink state
  - How shall a ClassMA seeing \( s \) know what happened since \( s \) occurred last time?
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- **Idea:** \( \mathcal{A} \) “colors” positions by ++, +, −, −− such that:
  - If an \( s \)-position has + the next \( s \)-position has − (and \( − \rightarrow + \))
  - If an \( s \)-position has + the next \( s \)-position in the same class has +

A little bit infinite?
### Proof sketch for (b) [Björklund, S 07]

- **Strictness:** RAs can not express (L1)
- Isn’t RegA ⊆ ClassMA obvious?
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  - If an \(s\)-position has \(-\) the next \(s\)-position in the same class has \(+\)

### Proof sketch for (b) (cont.)

- Of course: **if such a coloring exists, (L6) holds:** the next \(s\)-position is never the next \(s\)-position in the same class

\[
\begin{array}{cccccccc}
S & S & S & S & S & S & S & S \\
2 & 3 & 2 & 5 & 3 & 2 & 5 & 2 \\
3 & 2 & 3 & 2 & 3 & 2 & 3 & 2 \\
\end{array}
\]

- If (L6) holds such a coloring can be **constructed** by applying the following rules:
### Proof sketch for (b) [Björklund, S 07]

- **Strictness:** RAs can not express (L1)
- Isn’t $\text{RegA} \subseteq \text{ClassMA}$ obvious?
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  - The register automaton for (L6) only needs one state plus a sink state
  - How shall a ClassMA seeing $d$ know what happened since $s$ occurred last time?
- **Idea:** $\mathcal{A}$ “colors” positions by $\begin{bmatrix} +, +, -, -, - \end{bmatrix}$
  - such that:
    - If an $s$-position has $\begin{bmatrix} + \end{bmatrix}$ the next $s$-position has $\begin{bmatrix} - \end{bmatrix}$ (and $\begin{bmatrix} - \end{bmatrix}$ $\rightarrow$ $\begin{bmatrix} + \end{bmatrix}$)
    - If an $s$-position has $\begin{bmatrix} + \end{bmatrix}$ the next $s$-position in the same class has $\begin{bmatrix} + \end{bmatrix}$

### Proof sketch for (b) (cont.)

- Of course: **if such a coloring exists, (L6) holds:** the next $s$-position is never the next $s$-position in the same class
  - $\begin{bmatrix} 2 & 3 & 2 & 5 & 3 & 2 & 5 & 2 & 3 \end{bmatrix}$

- **If (L6) holds such a coloring can be constructed** by applying the following rules:
  1. If no other rule applies: assign $\begin{bmatrix} + \end{bmatrix}$ to the rightmost $s$ without upper color $\checkmark$
### Proof sketch for (b) [Björklund, S 07]

- **Strictness:** RAs cannot express (L1)
- Isn’t $\text{RegA} \subseteq \text{ClassMA}$ obvious?
- Not entirely, consider (L6): **No two successive prints by the same process**
  - The register automaton for (L6) only needs one state plus a sink state
  - How shall a ClassMA seeing $s_d$ know what happened since $s_d$ occurred last time?
- **Idea:** $A$ "colors" positions by $+$, $-$, $+$, $-$ such that:
  - If an $s$-position has $+$, the next $s$-position has $-$ (and $-$ → $+$)
  - If an $s$-position in the same class has $+$

### Proof sketch for (b) (cont.)

- Of course: if such a coloring exists, (L6) holds: the next $s$-position is never the next $s$-position in the same class

\[
\begin{array}{cccccccc}
S & S & S & S & S & S & S & S \\
2 & 3 & 2 & 5 & 3 & 2 & 5 & 3 \\
\end{array}
\]

- If (L6) holds such a coloring can be constructed by applying the following rules:
  1. If no other rule applies: assign $+$ to the rightmost $s$ without upper color
  2. Whenever $+$ is assigned to an $s$-position assign $-$ to its left $s$-neighbour and $+$ to the left $s$-neighbour in its class

A little bit infinite? Thomas Schwentick
### Proof sketch for (b) [Björklund, S 07]

- **Strictness:** RAs cannot express (L1)
- Isn’t \( \text{RegA} \subseteq \text{ClassMA} \) obvious?
- Not entirely, consider (L6): **No two successive prints by the same process**
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  - How shall a ClassMA seeing \( s_d \) know what happened since \( s_d \) occurred last time?

**Idea:** \( A \) “colors” positions by \(+, +, -, -\) such that:

- If an \( s \)-position has \( + \), the next \( s \)-position has \( - \)
- \( s \)-position has \( - \) (and \( - \) → \( + \))
- If an \( s \)-position has \( + \), the next \( s \)-position in the same class has \( + \)

### Proof sketch for (b) (cont.)

- **Of course:** if such a coloring exists, **(L6) holds:** the next \( s \)-position is never the next \( s \)-position in the same class

\[
\begin{array}{cccccccc}
& s_2 & s_3 & s_2 & s_5 & s_3 & s_2 & s_5 & s_3 \\
\hline
+ & + & + & - & + & - & + & - & + \\
\hline
\end{array}
\]

- **If (L6) holds** such a coloring can be constructed by applying the following rules:
  1. If no other rule applies: assign \( + \) to the rightmost \( s \) without upper color
  2. Whenever \( + \) is assigned to an \( s \)-position assign \( - \) to its left \( s \)-neighbour and \( + \) to the left \( s \)-neighbour in its class
  3. Whenever \( + \) is assigned to an \( s \)-position assign \( - \) to its right \( s \)-neighbour

A little bit infinite? Thomas Schwentick
**Class Memory Automata (4/5)**

<table>
<thead>
<tr>
<th>Proof sketch for (b) [Björklund, S 07]</th>
<th>Proof sketch for (b) (cont.)</th>
</tr>
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<tr>
<td>- Strictness: RAs can not express (L1)</td>
<td>- Of course: if such a coloring exists, (L6) holds: the next $s$-position is never the next</td>
</tr>
<tr>
<td>- Isn’t $\text{RegA} \subseteq \text{ClassMA}$ obvious?</td>
<td>$s$-position in the same class</td>
</tr>
<tr>
<td>- Not entirely, consider (L6): <strong>No two successive prints by the same process</strong></td>
<td>$s$</td>
</tr>
<tr>
<td>- The register automaton for (L6) only needs one state plus a sink state</td>
<td>$s$</td>
</tr>
<tr>
<td>- How shall a ClassMA seeing $s$ know what happened since $d$ occurred last time?</td>
<td>$s$</td>
</tr>
<tr>
<td>- <strong>Idea</strong>: $A$ “colors” positions by $\begin{array}{c} +, +, -, -, - \ +, -, +, - \end{array}$</td>
<td>$s$</td>
</tr>
<tr>
<td>- If an $s$-position has $\begin{array}{c} + \ + \end{array}$ the next $s$-position has</td>
<td>$s$</td>
</tr>
<tr>
<td>- (and $\begin{array}{c} - \ - \end{array} \rightarrow +$)</td>
<td>$s$</td>
</tr>
<tr>
<td>- If an $s$-position in the same class has $\begin{array}{c} + \ + \end{array}$</td>
<td>$s$</td>
</tr>
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A little bit infinite? Thomas Schwentick
Proof sketch for (b) [Björklund, S 07]

- **Strictness:** RAs can not express (L1)
- Isn’t RegA ⊆ ClassMA obvious?
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**Idea:** \( A \) “colors” positions by \(+\), \(-\), \(\mathbf{+}\), \(-\) such that:

- If an \( s \)-position has \(\mathbf{+}\) the next \( s \)-position has \(-\) (and \(-\) \(\rightarrow\) \(\mathbf{+}\))
- If an \( s \)-position has \(+\) the next \( s \)-position in the same class has \(+\)

Proof sketch for (b) (cont.)

- Of course: **if such a coloring exists, (L6) holds:** the next \( s \)-position is never the next \( s \)-position in the same class

\[
\begin{array}{cccccccc}
S & S & S & S & S & S & S & S \\
2 & 3 & 2 & 5 & 3 & 2 & 5 & 2 \\
\hline
+ & - & + & - & + & - & + & - \\
\hline
\end{array}
\]

- **If (L6) holds such a coloring can be constructed** by applying the following rules:
  1. If no other rule applies: assign \(\mathbf{+}\) to the rightmost \( s \) without upper color
  2. Whenever \(\mathbf{+}\) is assigned to an \( s \)-position assign \(-\) to its left \( s \)-neighbour and \(\mathbf{+}\) to the left \( s \)-neighbour in its class
  3. Whenever \(\mathbf{+}\) is assigned to an \( s \)-position assign \(-\) to its right \( s \)-neighbour \(\checkmark\)
Proof sketch for (b) [Björklund, S 07]

- **Strictness:** RAs can not express (L1)
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  - How shall a ClassMA seeing \(s^d\) know what happened since \(s^d\) occurred last time?
- **Idea:** \(A\) “colors” positions by \(\begin{array}{cccc}
+ & + & - & - \\
+ & - & + & - \\
\end{array}\)
  such that:
  - If an \(s\)-position has \(+\), the next \(s\)-position has \(-\) (and \(-\) → \(+\))
  - If an \(s\)-position in the same class has \(+\)

Proof sketch for (b) (cont.)

- Of course: **if such a coloring exists, (L6) holds:** the next \(s\)-position is never the next \(s\)-position in the same class
  \[
  \begin{array}{cccccccc}
  \text{s}_2 & \text{s}_3 & \text{s}_2 & \text{s}_5 & \text{s}_3 & \text{s}_2 & \text{s}_5 & \text{s}_2 \\
  + & - & + & + & + & - & - & + \\
  \end{array}
  \]
- **If (L6) holds such a coloring can be constructed** by applying the following rules:
  1. If no other rule applies: assign \(+\) to the rightmost \(s\) without upper color ✓
  2. Whenever \(+\) is assigned to an \(s\)-position assign \(\square\) to its left \(s\)-neighbour and \(\square\) to the left \(s\)-neighbour in its class
  3. Whenever \(+\) is assigned to an \(s\)-position assign \(\square\) to its right \(s\)-neighbour
Class Memory Automata (4/5)

Proof sketch for (b) [Björklund, S 07]

- Strictness: RAs can not express (L1)
- Isn’t \( \text{RegA} \subseteq \text{ClassMA} \) obvious?
- Not entirely, consider (L6): No two successive prints by the same process
  - The register automaton for (L6) only needs one state plus a sink state
  - How shall a ClassMA seeing \( s \) know what happened since \( d \) occurred last time?
- Idea: \( \mathcal{A} \) “colors” positions by +, −, +−, −+, such that:
  - If an \( s \)-position has + the next \( s \)-position has − (and − → +)
  - If an \( s \)-position has + the next \( s \)-position in the same class has +

Proof sketch for (b) (cont.)

- Of course: if such a coloring exists, (L6) holds: the next \( s \)-position is never the next \( s \)-position in the same class

\[
\begin{array}{cccccccc}
\text{s2} & \text{s3} & \text{s2} & \text{s3} & \text{s2} & \text{s3} & \text{s2} \\
+ & − & + & + & + & + & − \\
− & + & + & − & − & − & + \\
\end{array}
\]

- If (L6) holds such a coloring can be constructed by applying the following rules:
  1. If no other rule applies: assign + to the rightmost \( s \) without upper color
  2. Whenever + is assigned to an \( s \)-position assign − to its left \( s \)-neighbour and + to the left \( s \)-neighbour in its class
  3. Whenever + is assigned to an \( s \)-position assign − to its right \( s \)-neighbour

A little bit infinite? Thomas Schwentick
Proof sketch for (b) [Björklund, S 07]

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- Isn’t RegA $\subseteq$ ClassMA obvious?
- Not entirely, consider (L6): No two successive prints by the same process
  - The register automaton for (L6) only needs one state plus a sink state
  - How shall a ClassMA seeing $s_d$ know what happened since $s_d$ occurred last time?

- Idea: $\mathcal{A}$ “colors” positions by $+, +-, -, -+$ such that:
  - If an $s$-position has $+$ the next $s$-position has $-$ (and $-$ $\rightarrow$ $+$)
  - If an $s$-position has $+$ the next $s$-position in the same class has $+$

Proof sketch for (b) (cont.)

- Of course: if such a coloring exists, (L6) holds: the next $s$-position is never the next $s$-position in the same class

- If (L6) holds such a coloring can be constructed by applying the following rules:
  1. If no other rule applies: assign $+$ to the rightmost $s$ without upper color
  2. Whenever $+$ is assigned to an $s$-position assign $-$ to its left $s$-neighbour and $+$ to the left $s$-neighbour in its class
  3. Whenever $+$ is assigned to an $s$-position assign $+$ to its right $s$-neighbour ✓

A little bit infinite? Thomas Schwentick
Proof sketch for (b) [Björklund, S 07]

- **Strictness**: RAs can not express (L1)
- Isn’t $\text{RegA} \subseteq \text{ClassMA}$ obvious?
- Not entirely, consider (L6): **No two successive prints by the same process**
  - The register automaton for (L6) only needs one state plus a sink state
  - How shall a ClassMA seeing $s_d$ know what happened since $s_d$ occurred last time?
- **Idea**: $\mathcal{A}$ “colors” positions by $\begin{array}{cccccc}
+ & + & - & - & - \\
+ & - & + & - & - \\
\end{array}$
such that:
  - If an $s$-position has $\begin{array}{c}
+
\end{array}$ the next $s$-position has $\begin{array}{c}
-
\end{array}$ (and $\begin{array}{c}
-
\end{array} \rightarrow \begin{array}{c}
+
\end{array}$)
  - If an $s$-position has $\begin{array}{c}
+
\end{array}$ the next $s$-position in the same class has $\begin{array}{c}
+
\end{array}$

Proof sketch for (b) (cont.)

- Of course: **if such a coloring exists, (L6) holds**: the next $s$-position is never the next $s$-position in the same class
  - $\begin{array}{cccccccc}
S & S & S & S & S & S & S & S \\
\begin{array}{cc}
+ & - \\
- & + \\
\end{array} & \begin{array}{cc}
- & + \\
+ & - \\
\end{array} & \begin{array}{cc}
+ & - \\
- & + \\
\end{array} & \begin{array}{cc}
- & + \\
+ & - \\
\end{array} & \begin{array}{cc}
+ & - \\
- & + \\
\end{array} & \begin{array}{cc}
- & + \\
+ & - \\
\end{array} & \begin{array}{cc}
+ & - \\
- & + \\
\end{array} & \begin{array}{cc}
- & + \\
+ & - \\
\end{array} \\
\end{array}$
- **If (L6) holds such a coloring can be constructed** by applying the following rules:
  1. If no other rule applies: assign $\begin{array}{c}
+
\end{array}$ to the rightmost $s$ without upper color
  2. Whenever $\begin{array}{c}
+
\end{array}$ is assigned to an $s$-position assign $\begin{array}{c}
-
\end{array}$ to its left $s$-neighbour and $\begin{array}{c}
+
\end{array}$ to the left $s$-neighbour in its class
  3. Whenever $\begin{array}{c}
+
\end{array}$ is assigned to an $s$-position assign $\begin{array}{c}
-
\end{array}$ to its right $s$-neighbour
Proof sketch for (b) [Björklund, S 07]

- **Strictness**: RAs can not express (L1)
- **Isn’t** $\text{RegA} \subseteq \text{ClassMA}$ **obvious?**
- **Not entirely**, consider (L6): **No two successive prints by the same process**
  - The register automaton for (L6) only needs one state plus a sink state
  - How shall a ClassMA seeing $s_d$ know what happened since $s_d$ occurred last time?

**Idea**: $A$ “colors” positions by $\text{++, +, −, −−}$ such that:

- If an $s$-position has $+$ the next $s$-position has $+$ (and $-$ $\rightarrow$ $+$)
- If an $s$-position in the same class has $+$

Proof sketch for (b) (cont.)

**Of course**: if such a coloring exists, (L6) **holds**: the next $s$-position is never the next $s$-position in the same class

$$\begin{align*}
S & \quad S & \quad S & \quad S & \quad S & \quad S & \quad S & \quad S \\
2 & \quad 3 & \quad 2 & \quad 5 & \quad 3 & \quad 2 & \quad 5 & \quad 2 \\
\text{---} & \quad \text{---} & \quad \text{---} & \quad \text{---} & \quad \text{---} & \quad \text{---} & \quad \text{---} & \quad \text{---}
\end{align*}$$

- **If (L6) holds** such a coloring can be **constructed** by applying the following rules:
  1. If no other rule applies: assign $+$ to the rightmost $s$ without upper color
  2. Whenever $+$ is assigned to an $s$-position assign $-$ to its left $s$-neighbour and $+$ to the left $s$-neighbour in its class
  3. Whenever $+$ is assigned to an $s$-position assign $-$ to its right $s$-neighbour $\checkmark$
Class Memory Automata (4/5)

Proof sketch for (b) [Björklund, S 07]

- Strictness: RAs can not express (L1)
- Isn’t RegA ⊆ ClassMA obvious?
- Not entirely, consider (L6): **No two successive prints by the same process**
  - The register automaton for (L6) only needs one state plus a sink state
  - How shall a ClassMA seeing $s_d$ know what happened since $s_d$ occurred last time?
- **Idea:** $A$ “colors” positions by $\begin{array}{c} +, +, -, -, \end{array}$ such that:
  - If an $s$-position has $\begin{array}{c} + \end{array}$ the next $s$-position has $\begin{array}{c} (and \begin{array}{c} - \end{array} \rightarrow \begin{array}{c} + \end{array}) \end{array}$
  - If an $s$-position has $\begin{array}{c} + \end{array}$ the next $s$-position in the same class has $\begin{array}{c} + \end{array}$

Proof sketch for (b) (cont.)

- Of course: **if such a coloring exists, (L6) holds:** the next $s$-position is never the next $s$-position in the same class

\[
\begin{array}{cccccccc}
S & S & S & S & S & S & S & S \\
2 & 3 & 2 & 5 & 3 & 2 & 5 & 3 \\
\hline
+ & + & - & - & + & + & - & - \\
\end{array}
\]

- If (L6) holds such a coloring can be constructed by applying the following rules:
  1. If no other rule applies: assign $\begin{array}{c} + \end{array}$ to the rightmost $s$ without upper color
  2. Whenever $\begin{array}{c} + \end{array}$ is assigned to an $s$-position assign $\begin{array}{c} + \end{array}$ to its left $s$-neighbour and $\begin{array}{c} + \end{array}$ to the left $s$-neighbour in its class
  3. Whenever $\begin{array}{c} + \end{array}$ is assigned to an $s$-position assign $\begin{array}{c} + \end{array}$ to its right $s$-neighbour

A little bit infinite? Thomas Schwentick
Proof sketch for (b) [Björklund, S 07]

- **Strictness:** RAs can not express (L1)
- **Isn’t** $\text{RegA} \subseteq \text{ClassMA}$ **obvious?**
- **Not entirely,** consider (L6): **No two successive prints by the same process**
  - The register automaton for (L6) only needs one state plus a sink state
  - How shall a ClassMA seeing $s_d$ know what happened since $s_d$ occurred last time?
- **Idea:** $\mathcal{A}$ “colors” positions by $s_d$ such that:
  - If an $s$-position has $\color{red}{+}$ the next $s$-position has $\color{blue}{-}$ (and $\color{blue}{-} \rightarrow \color{red}{+}$)
  - If an $s$-position has $\color{red}{+}$ the next $s$-position **in the same class** has $\color{red}{+}$

Proof sketch for (b) (cont.)

- **Of course:** if such a coloring exists, (L6) **holds:** the next $s$-position is never the next $s$-position in the same class
  - $s_2 s_3 s_2 s_5 s_3 s_2 s_5 s_2 s_3$

- **If** (L6) holds such a coloring can be **constructed** by applying the following rules:
  1. If no other rule applies: assign $\color{red}{+}$ to the rightmost $s$ without upper color
  2. Whenever $\color{red}{+}$ is assigned to an $s$-position assign $\color{blue}{-}$ to its left $s$-neighbour and $\color{red}{+}$ to the left $s$-neighbour in its class
  3. Whenever $\color{red}{+}$ is assigned to an $s$-position assign $\color{blue}{-}$ to its right $s$-neighbour $\checkmark$
Proof sketch for (b) [Björklund, S 07]

- Strictness: RAs can not express (L1)
- Isn’t $\text{RegA} \subseteq \text{ClassMA}$ obvious?
- Not entirely, consider (L6): No two successive prints by the same process
  - The register automaton for (L6) only needs one state plus a sink state
  - How shall a ClassMA seeing \text{s} know what happened since \text{d} occurred last time?

**Idea:** $\mathcal{A}$ “colors” positions by ++, +, −, −− such that:

- If an \text{s}-position has + the next \text{s}-position has + (and − → +)
- If an \text{s}-position in the same class has +

Proof sketch for (b) (cont.)

- Of course: if such a coloring exists, (L6) holds: the next \text{s}-position is never the next \text{s}-position in the same class

<p>| | | | | | | |</p>
<table>
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<td>\text{s}</td>
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</tbody>
</table>

- If (L6) holds such a coloring can be constructed by applying the following rules:
  1. If no other rule applies: assign + to the rightmost \text{s} without upper color
  2. Whenever + is assigned to an \text{s}-position assign − to its left \text{s}-neighbour and + to the left \text{s}-neighbour in its class
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A little bit infinite? Thomas Schwentick
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- **Idea**: \( \mathcal{A} \) “colors” positions by \( ++, +-, -, +, -- \) such that:
  - If an \( s \)-position has \( + \) the next \( s \)-position has \( - \) (and \( - \) \( \rightarrow \) \( + \))
  - If an \( s \)-position has \( + \) the next \( s \)-position in the same class has \( + \)

Proof sketch for (b) (cont.)

- Of course: **if such a coloring exists, (L6) holds**: the next \( s \)-position is never the next \( s \)-position in the same class

\[
\begin{array}{cccccccc}
S & S & S & S & S & S & S & S \\
\underline{+} & - & + & + & - & + & + & + \\
- & + & - & + & + & - & - & - \\
\end{array}
\]

- If (L6) holds such a coloring can be constructed by applying the following rules:
  1. If no other rule applies: assign \( + \) to the rightmost \( s \) without upper color
  2. Whenever \( + \) is assigned to an \( s \)-position assign \( - \) to its left \( s \)-neighbour and \( + \) to the left \( s \)-neighbour in its class
  3. Whenever \( + \) is assigned to an \( s \)-position assign \( + \) to its right \( s \)-neighbour

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**Proof sketch for (b) [Björklund, S 07]**

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**Idea:** \( \mathcal{A} \) “colors” positions by \(+, +, -, -\) such that:

- If an \( s \)-position has \(+\) the next \( s \)-position has \(-\) (and \(-\) \(\rightarrow\) \(+\))
- If an \( s \)-position has \(-\) the next \( s \)-position in the same class has \(+\)

**Proof sketch for (b) (cont.)**

- Of course: **if such a coloring exists, (L6) holds:** the next \( s \)-position is never the next \( s \)-position in the same class

\[
\begin{array}{cccccccc}
  S_2 & S_3 & S_2 & S_5 & S_3 & S_2 & S_5 & S_3 \\
  + & + & - & + & - & + & - & + \\
 - & + & - & - & + & - & - & - \\
\end{array}
\]

- **If (L6) holds such a coloring can be constructed** by applying the following rules:
  1. If no other rule applies: assign \(+\) to the rightmost \( s \) without upper color
  2. Whenever \(+\) is assigned to an \( s \)-position assign \(-\) to its left \( s \)-neighbour and \(+\) to the left \( s \)-neighbour in its class
  3. Whenever \(+\) is assigned to an \( s \)-position assign \(-\) to its right \( s \)-neighbour
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  - If an $s$-position has $+$ the next $s$-position in the same class has $+$

### Proof sketch for (b) (cont.)

- Of course: if such a coloring exists, (L6) **holds:** the next $s$-position is never the next $s$-position in the same class

```
S  2  S  3  S  2  S  3  S  2  S  3  S  2  S  3  S  3
+ + - + - - + + - - - + +
- - - + + - - + + - - -
```
- If (L6) holds such a coloring can be constructed by applying the following rules:
  1. If no other rule applies: assign $+$ to the rightmost $s$ without upper color
  2. Whenever $+$ is assigned to an $s$-position assign $-$ to its left $s$-neighbour and $+$ to the left $s$-neighbour in its class
  3. Whenever $+$ is assigned to an $s$-position assign $+$ to its right $s$-neighbour
- General proof of (b): similar coloring trick
<table>
<thead>
<tr>
<th></th>
<th>RegisterA</th>
<th>PebbleA</th>
<th>ClassMA</th>
<th>DClassMA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expressiveness</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(L2), (L6), (L7)</td>
<td>(L1)–(L7)</td>
<td>(L1)–(L7)</td>
<td>(L1)–(L5), (L7)</td>
<td></td>
</tr>
<tr>
<td><strong>Decidability</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-emptiness</td>
<td>✓</td>
<td>–</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Containment</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td><strong>Efficiency</strong></td>
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<tr>
<td>Data complexity word pr.</td>
<td>✓</td>
<td>✓</td>
<td>–</td>
<td>✓</td>
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<td><strong>Closure properties</strong></td>
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<tr>
<td>Union</td>
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<td>–</td>
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<td>Intersection</td>
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<td>✓</td>
<td>✓</td>
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<td>Complement</td>
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<td>✓</td>
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<td><strong>Robustness</strong></td>
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<td>-</td>
<td>✓</td>
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</tbody>
</table>
Inclusion structure of Automata Models

A little bit infinite? Thomas Schwentick

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Alternating Register Automata (1/2)

- How to turn register automata into a reasonably strong, robust and decidable model?
How to turn register automata into a reasonably strong, robust and decidable model?

- 1N-RA are pretty weak
- 2D-RA are undecidable
How to turn register automata into a reasonably strong, robust and decidable model?

- 1N-RA are pretty weak
- 2D-RA are undecidable

[Demri, Lazić 06]:
- Alternating one-way register automata with one register: $\text{ARA}_1$
Alternating Register Automata (1/2)

- How to turn register automata into a reasonably strong, robust and decidable model?
  - 1N-RA are pretty weak
  - 2D-RA are undecidable

- [Demri, Lazić 06]:
  - Alternating one-way register automata with one register: ARA₁

Theorem 5 [Demri, Lazić 06]

(a) Non-emptiness (and Containment) of ARA₁ on strings is decidable but not primitive recursive
Alternating Register Automata (1/2)

• How to turn register automata into a reasonably strong, robust and decidable model?
  ▶ 1N-RA are pretty weak
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Theorem 5 [Demri, Lazić 06]

(a) Non-emptiness (and Containment) of ARA₁ on strings is decidable but not primitive recursive

(b) Non-emptiness of ARA₁ on ω-strings is undecidable (even with Muller acceptance)
How to turn register automata into a reasonably strong, robust and decidable model?

- 1N-RA are pretty weak
- 2D-RA are undecidable

[Demri, Lazić 06]:
- Alternating one-way register automata with one register: $\text{ARA}_1$

Theorem 5 [Demri, Lazić 06]

(a) Non-emptiness (and Containment) of $\text{ARA}_1$ on strings is decidable but not primitive recursive

(b) Non-emptiness of $\text{ARA}_1$ on $\omega$-strings is undecidable (even with Muller acceptance)

- $\text{ARA}_1$ can express all properties (L1)-(L7)
- $\text{ARA}_1$ can not remember two data values at a time
Alternating Register Automata (1/2)

- How to turn register automata into a reasonably strong, robust and decidable model?
  - 1N-RA are pretty weak
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(a) Non-emptiness (and Containment) of ARA$_1$ on strings is decidable but not primitive recursive

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- Safety ARA$_1$ reject only in the finite (and their complement languages are closed under adding suffixes)

- ARA$_1$ can express all properties (L1)-(L7)
- ARA$_1$ can not remember two data values at a time
## Alternating Register Automata (1/2)

- How to turn register automata into a reasonably strong, robust and decidable model?
  - 1N-RA are pretty weak
  - 2D-RA are undecidable

- [Demri, Lazić 06]:
  - Alternating one-way register automata with **one register**: $\text{ARA}_1$

### Theorem 5 [Demri, Lazić 06]

- (a) Non-emptiness (and Containment) of $\text{ARA}_1$ on strings is decidable but not primitive recursive
- (b) Non-emptiness of $\text{ARA}_1$ on $\omega$-strings is undecidable (even with Muller acceptance)

- $\text{ARA}_1$ can express all properties (L1)-(L7)
- $\text{ARA}_1$ can not remember two data values at a time

- **Safety $\text{ARA}_1$** reject only in the finite (and their complement languages are closed under adding suffixes)

### Theorem 6 [Lazić 06]

- (a) Non-emptiness of safety $\text{ARA}_1$ on $\omega$-strings is $\text{EXPSPACE}$-complete
- (b) Containment of safety $\text{ARA}_1$ on $\omega$-strings is decidable but not primitive recursive
### Alternating Register Automata (2/2)

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<thead>
<tr>
<th></th>
<th>RegisterA</th>
<th>PebbleA</th>
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<th>DClassMA</th>
<th>ARA(_1)</th>
<th>Safe ARA(_1)</th>
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<td></td>
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</tr>
<tr>
<td></td>
<td>(L2),(L6),(L7)</td>
<td>(L1)–(L7)</td>
<td>(L1)–(L7)</td>
<td>(L1)–(L5),(L7)</td>
<td>(L1)–(L7)</td>
<td>(L1),(L4),(L6)</td>
</tr>
<tr>
<td><strong>Decidability</strong></td>
<td>✓</td>
<td>–</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>Non-emptiness</td>
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<tr>
<td>Containment</td>
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<td><strong>Efficiency</strong></td>
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<tr>
<td><strong>Closure properties</strong></td>
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<td>Union</td>
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<td>Intersection</td>
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<td><strong>Robustness</strong></td>
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Logic

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  - Temporal Logics
- Other Models
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A little bit infinite? Thomas Schwentick
Logics for Data Strings/Trees

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<td>$\uparrow 1$</td>
<td>successor relation on positions</td>
<td>$E_{\rightarrow}$ horizontal neighbor (&quot;next sibling&quot;)</td>
</tr>
<tr>
<td>$&lt;$</td>
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<td>$E_{\rightarrow}$ transitive closure of $E_{\rightarrow}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$E_{\downarrow}$ transitive closure of $E_{\downarrow}$</td>
</tr>
<tr>
<td>$\sim$</td>
<td>$x \sim y$ if positions $x$ and $y$ have the same $D$-value</td>
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### Logical language...

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<tr>
<td>$E \downarrow$</td>
<td>Parent-child</td>
</tr>
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<td>Transitive closure of $E \rightarrow$</td>
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- Of course: $\sim$ *is an equivalence relation*
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| \(+1\)             | successor relation on positions | \( E\rightarrow \) | horizontal neighbor  
  ("next sibling") |
| \(<\)              | order relation on positions | \( E\downarrow \) | parent-child |
| \( \sim \)         | \( x \sim y \) if positions \( x \) and \( y \) have the same \( D\)-value | \( \sim \) | \( x \sim y \) if nodes \( x \) and \( y \) have the same \( D\)-value |
| \( \pm1 \)         | next position in the same class |

- Of course: \( \sim \) **is an equivalence relation**
- No other operations on data values, in particular no arithmetic!
We know:

- First-order logic is undecidable in general
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- First-order logic is decidable on strings
A first attempt

- We know:
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- What about First-order logic on data strings?
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What about First-order logic on data strings?

Theorem 7 [Bojańczyk et al. 06a]

- Satisfiability of First-Order formulas on data strings is undecidable, even for formulas with 3 variables

Proof idea

- Reduction from PCP:
  - Given: $(u_1, v_1), \ldots, (u_k, v_k)$, pairs of strings
  - Question: is there a sequence $i_1, \ldots, i_n$ such that $u_{i_1} \cdots u_{i_n} = v_{i_1} \cdots u_{i_n}?$

A little bit infinite?
A first attempt

- We know:
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A bit more detail

- Encode solution candidates as data strings over \(\{a, b, \#, 1, \ldots, k\}\) of the form \(u \# v\)
- Each occurrence of a \(u_i\) is prefixed by \(i\): E.g., if \(u_1 = aba\) and \(u_2 = bb\) then \(121\) is encoded by \(1aba2bb1aba\)
- Each data value occurs exactly twice, once in \(u\) and once in \(v\)
  - corresponding positions should have the same data value (and same number/symbol)
- Crucial: check that the sequence of data values is the same on both sides for number positions and letter positions
  - Important subformula:
    \[
    x \sim y \rightarrow \exists z \,(x + 1 = z \land \exists x\, y + 1 = x \land z \sim x)
    \]
  - "if \(x\) and \(y\) are equivalent then their right neighbors are also equivalent"
Two Variables on Data Strings: A Useful Restriction?

- A classical approach: Restriction to 2 variables
- Does this restriction give us anything useful?
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- A classical approach: Restriction to 2 variables
- Does this restriction give us anything useful?
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  2. lot of useful properties can be expressed with only two variables

### Examples

(L1) No two $a$-positions do have the same data value
$$\forall x \forall y (x \sim y \land a(x) \land a(y)) \rightarrow x = y$$

(L2) There are two $a$-positions with the same data value
$$\exists x \exists y x \sim y \land a(x) \land a(y) \land x \neq y$$

(L3) For each $a$-position there is a $b$-position with the same data value
$$\forall x \exists y a(x) \rightarrow (b(y) \land x \sim y)$$

(L4) A print job of a user has to be printed before the next one can be requested
$$\forall x \forall y y = x \pm 1 \rightarrow [(r(x) \rightarrow s(s)) \land (s(x) \rightarrow t(y))]$$

(L5) Each print request of a user is eventually followed by a print
$$\forall x \exists y r(x) \rightarrow (s(y) \land x < y \land x \sim y)$$

(L6) Between two successive print jobs of the same user some other user’s job has to be printed
not expressible

(L7) After each printed job a job of some other user is eventually printed
$$\forall x \exists y r(x) \rightarrow (s(y) \land x < y \land x \not\sim y)$$
### Example

- $\varphi_a$:
  - $\forall x \forall y (x \sim y \land a(x) \land a(y)) \rightarrow x = y$
  - all $a$'s are in different classes
- Similarly: $\varphi_b$
- $\psi_{a,b}$:
  - $\psi_{a,b} = \forall x \exists y (a(x) \rightarrow (b(y) \land x \sim y))$
  - each class with an $a$ also contains a $b$
- Similarly: $\psi_{b,a}$.
On the expressive power of $\text{FO}^2$ on data strings (1/2)

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$\varphi = \varphi_a \land \varphi_b \land \psi_{a,b} \land \psi_{b,a}$ implies:
the numbers of $a$ and $b$-labeled positions are equal

- In a similar fashion: number of $a$'s, $b$'s and $c$'s are equal

A little bit infinite? Thomas Schwentick
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- In a similar fashion: number of $a$'s, $b$'s and $c$'s are equal

$\Rightarrow$ The string projection of an $\text{FO}^2$-definable data language need not be context-free
More example properties

- Let $\alpha$ and $\beta$ denote unary quantifier-free formulas ("types")
- $\text{FO}^2$ can express
  
  ▶
  ▶
  ▶
  ▶
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  ▶
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On the expressive power of $\text{FO}^2$ on data strings (2/2)

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</tr>
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<tr>
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</tr>
<tr>
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Theorem 8 [Bojańczyk et al. 06a]

Satisfiability of $\text{FO}^2(\sim, <, +1, \neq 1)$ on data strings is decidable
We transform two-variable formulas into satisfiability equivalent formulas of **existential monadic second-order logic**

**“Scott normal form”**: \( \exists R_1, \ldots, R_k \forall x \forall y \chi \land \bigwedge_i \forall x \exists y \chi_i \)

**Intermediate normal form**:
\[
\exists R_1 \cdots R_m \theta_1 \land \cdots \land \theta_n
\]

\( \theta_i \):

1. \( \forall x \forall y \ (\delta(x, y) \geq 2 \land \alpha(x) \land \beta(y) \land \begin{array}{c} x \sim y \\ x \not\sim y \end{array}) \rightarrow \begin{array}{l} x < y \\ x > y \end{array} \)

2. \( \forall x \exists y \ \alpha(x) \rightarrow (\beta(y) \land \begin{array}{c} x + 1 < y \\ x + 1 = y \\ x = y \\ x = y + 1 \\ x > y + 1 \end{array} \land \begin{array}{c} x \sim y \\ x \not\sim y \end{array}) \)

3. \( \forall x \forall y \ \theta \quad (\theta \text{ quantifier-free, DNF, no } \sim) \)

Both steps are straightforward
### Data normal form & Class Memory Automata

- **Data normal form:**
  - Disjunction of formulas: \( \exists R_1 \cdots R_n \, \theta_1 \wedge \cdots \wedge \theta_n \)
  - \( \theta_i \):
    - (a) data-blind
    - (b) Each class contains at most one \( \alpha \)
    - (c) In each class, every \( \alpha \) occurs before every \( \beta \)
    - (d) Each class with an \( \alpha \) also has a \( \beta \)
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## Data normal form & Class Memory Automata

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### Final Step:
- Each $\theta_i$ can be recognized by a Class Memory Automaton
- Existential monadic quantification corresponds to nondeterminism in CMAs
- CMAs are closed under union and intersection
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Data normal form & Class Memory Automata

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**Decidability of $\text{FO}^2(\sim, <, +1, \pm 1)$ follows from decidability of Non-emptiness for Class Memory Automata**

**Corollary:** $\text{ClassMA} \equiv \text{EMSO}^2(\sim, <, +1, \pm 1)$
FO$^2$ on Data Strings: Complexity

- Complexitywise, Satisfiability of $\text{FO}^2(\sim, <, +1)$ is basically equivalent to Non-Emptiness of multicounter automata
  
  $\Rightarrow$ Unknown complexity
$\text{FO}^2$ on Data Strings: Complexity

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- **Restrictions:**
  - $\text{FO}^2(\sim, <)$: complete for $\text{NEXPTIME}$ [David 04]
  - $\text{FO}^2(\sim, +1)$: in $3\text{NEXPTIME}$ [Bojańczyk et al. 06b]
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  - \( \text{FO}^2(\sim, +1) \): in \( 3\text{NEXPTIME} \) [Bojańczyk et al. 06b]

- Extensions:
  - \(+2, +3, \ldots\): same results
  - \( \omega \)-strings: same results
  - Linear order on data values: undecidable
### Theorem 9 [Bojańczyk et al. 06b]

For any vector addition tree automaton $A$, a formula $\varphi_A \in FO^2(\sim, <, +1)$ can be computed such that:

$L(A) \neq \emptyset$ iff $\varphi_A$ has a model
Theorem 9 [Bojańczyk et al. 06b]

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- Decidability of emptiness of vector addition tree automata is an open problem
- It is equivalent to decidability of Multiplicative Exponential Linear Logic

We concentrate on $FO^2(\sim, +1)$
# Two-Variable Logic on Data Trees

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**Theorem 10** [Bojańczyk et al. 06b]

Satisfiability of $FO^2(\sim, +1)$ on data trees is decidable
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### Theorem 10 [Bojańczyk et al. 06b]

Satisfiability of $\text{FO}^2(\sim, +1)$ on data trees is decidable

- The intermediate steps of the proof are similar as for data strings
- But additional techniques needed:
  - Model normalization by cut-and-paste arguments
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## Two-Variable Logic on Data Trees

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**Complexity:**

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- On trees of bounded depth: $\text{FO}^2$ with all axes decidable [Björklund, Bojańczyk 07]
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• More results on reasoning about XML integrity constraints:
  [Arenas et al. 05]
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▷ Temporal Logics

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A little bit infinite? Thomas Schwentick
Temporal Logics and the Freeze Quantifier

- $\text{FO}^2$ is natural to consider from an XML point of view
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| (1) undecidable in general                                                        |
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LTL with Freeze vs. $\text{FO}^2$

- LTL with Freeze cannot express:
  - (L3) for each $a$-position there is a $b$-position with the same data value
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$\Rightarrow$ LTL with Freeze and $\text{FO}^2$ are incomparable
LTL with Freeze: Extensions and Restrictions

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  - $X^{-1}, G^{-1}, F^{-1}, U^{-1}$
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LTL with Freeze: Extensions and Restrictions

- **LTL with Freeze and past modalities:** [Demri, Lazić 06]
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  - Finitary and Infinitary Satisfiability are decidable
# Automata and Logics

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LTL on top of first-order logic
- [Spielmann 00]: Verification of relational transducers
- [Abdulla et al. 04]: ...even on top of MSO
- [Deutsch et al. 04]: Verification of web services
- In all cases: restricted comparison of data values of different states
Some Related Work on Data Trees

[Kaminski, Tan 06] Register automata for trees

[Jurdziński, Lazić 07]

- Alternation-free modal $\mu$-calculus
  - Basically identical results as for LTL with Freeze
  - In particular:
    - Computationally equivalent to Incrementing Tree Counter Automata
    - Safety fragment decidable
- Alternating Automata
- XPath satisfiability
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Data strings:
- Attracted most attention so far
- No obvious analogon of regular languages (so far)
- But “logic $\rightarrow$ automaton $\rightarrow$ analysis” possible to some extent
- Applicability in Verification has yet to be explored:
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  - Its ability to talk about data values is limited (e.g., no arithmetic)
    - Is it really useful?
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Conclusion

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- In both cases we need:
  - Models with better complexity
  - Models with richer data access
Technical Questions:

- Precise complexity of Satisfiability of $\mathbf{FO}^2(\sim, +1)$ on data strings
- Precise complexity of Satisfiability of $\mathbf{FO}^2(\sim, +1)$ on data trees
- Is Satisfiability of $\mathbf{FO}^2(\sim, <, +1)$ on data trees decidable?
- Upper complexity bounds for Satisfiability of $\mathbf{FO}^2(\sim, <, +1, \pm 1)$ on data strings
Open Problems

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To be explored:
- Is there a generic class of regular data (string/tree) languages?
- Find models with better complexities
- Study the trade-off between more expressive data access and complexity/decidability
- Find larger decidable fragments of data-aware XPath
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