Fortgeschrittene Themen der Wissensrepräsentation

Gabriele Kern-Isberner
LS 1 – Information Engineering

TU Dortmund
Sommersemester 2017
Kapitel 5

5. Das P-Axiom in der Wissensrevision

[GKI & Brewka, IJCAI 2017]
5. Das P-Axiom in der Wissensrevision

5.1 Einführung
**Problem:** The following revision is compatible with classical AGM belief revision theory:

\[ K = Cn(A, B) \text{ with } \Sigma(A) \cap \Sigma(B) = \emptyset, \text{ then } K \ast \neg A = Cn(\neg A, \neg B). \]

**Parikh’s solution (P):** If the language can be split into two disjoint parts and the new information affects just one part, then only this part should be changed.

**Peppas et al.’s extension:** Distinguishing between weak and strong (P), characterizing families of total preorders that satisfy strong (P) (on the propositional level!).
Main contributions

- Extending the notion of syntax splitting and strong (P) to total preorders/epistemic states for iterated revision
- Sharpening the results for the framework of Spohn’s ranking functions
- Presenting a revision operator satisfying strong (P)
Parikh’s axiom (P)\(^1\):

\((P)\) If \(K = Cn(A, B)\) with \(\Sigma(A) \cap \Sigma(B) = \emptyset\), and \(C \in \mathcal{L}(A)\), then 
\[K * C = Cn((Cn_{\Sigma(A)}(A) \circ C) \cup \{B\}),\]
where \(\circ\) is a revision operator of \(\mathcal{L}(\Sigma(A))\).

Peppas et al.’s version of strong (P) \(^2\):

\(\text{(R1)}\) If \(K = Cn(A, B)\), \(\Sigma(A) \cap \Sigma(B) = \emptyset\), and \(C \in \mathcal{L}(A)\) then 
\[\overline{(K * C) \cap \mathcal{L}(\Sigma(A))} = K \cap \mathcal{L}(\Sigma(A)).\]

\(\text{(R2)}\) If \(K = Cn(A, B)\), \(\Sigma(A) \cap \Sigma(B) = \emptyset\), and \(C \in \mathcal{L}(A)\), then 
\[\overline{(K * C) \cap \mathcal{L}(\Sigma(A))} = (Cn(A) * C) \cap \mathcal{L}(\Sigma(A)).\]

\(^1[\text{Parikh, 1999}]\)
\(^2[\text{Peppas et al., 2015}]\)
Theorem 1 (Peppas et al., 2015, Theorem 6)

Let $\star$ be an AGM revision operator, and let $\{\preceq^K\}_K$ be a family of faithful preorders (one for each consistent belief set $K$, corresponding to $\star$). Then $\star$ satisfies (R1) and (R2) (i.e., strong (P)) iff $\{\preceq^K\}_K$ satisfies (Q1) - (Q3).
Syntax splitting according to [Parikh, 1999; Peppas et al., 2015]:

Let \((\Sigma_1, \ldots, \Sigma_n)\) be a partition of \(\Sigma\), i.e., \(\Sigma = \Sigma_1 \cup \ldots \cup \Sigma_n\), and \(\Sigma_i \cap \Sigma_j = \emptyset\) for \(i \neq j\).

A belief set \(K\) splits over \((\Sigma_1, \ldots, \Sigma_n)\) iff there are propositions \(\phi_i \in \mathcal{L}(\Sigma_i), i = 1, \ldots, n\), such that \(K = Cn(\phi_1, \ldots, \phi_n)\).

\((\Sigma_1, \ldots, \Sigma_n)\) is then called a \(K\)-splitting.
Charakterisierung von strong (P) (Forts.)

Let a theory $K$ be given, and let $\mathcal{F} = (F_i)_{i \in I}$ be its unique finest $K$-splitting, i.e., for every other $K$-splitting $(\Sigma_j)_{j \in J}$, each $F_i$ is contained in some $\Sigma_j$.

Then for a world $\omega$,

$$\text{Diff}(K, \omega) = \bigcup \{ F_i \in \mathcal{F} \mid \text{for some } \phi \in \mathcal{L}(F_i), K \models \phi \text{ and } \omega \models \neg \phi \}$$

For two worlds $\omega, \omega'$, let $\text{diff}(\omega, \omega')$ denote the set of atoms that have different truth values in the two worlds.

For a (contingent) proposition $A$ and a world $\omega$, $\omega^A$ is the restriction of $\omega$ to $\mathcal{L}(A)$.

For a preorder $\preceq$ on $\Omega$, the $A$-filtering $\preceq^A$ of $\preceq$ is defined by $\omega \preceq^A \omega'$ iff there is $\omega_1 \models \omega^A$ such that for all $\omega'_1 \models \omega'^A$, $\omega_1 \preceq \omega'_1$. 
Charakterisierung von strong (P) (Forts.)

(Q1) If $\text{Diff}(K, \omega) \subset \text{Diff}(K, \omega')$ and $\text{diff}(\omega, \omega') \cap \text{Diff}(K, \omega) = \emptyset$, then $\omega \prec \omega'$.

(Q2) If $\text{Diff}(K, \omega) = \text{Diff}(K, \omega')$ and $\text{diff}(\omega, \omega') \cap \text{Diff}(K, \omega) = \emptyset$, then $\omega \approx \omega'$.

(Q3) If $K = \text{Cn}(A, B)$ and $\Sigma(A) \cap \Sigma(B) = \emptyset$, then $\preceq^A_K = \preceq^A_{\text{Cn}(A)}$. 

5. Das P-Axiom in der Wissensrevision

5.2 Syntax Splitting für die iterierte Revision
Sublanguages of $\mathcal{L}$

Epistemic states $\Psi$ are represented by a total preorder $\preceq_\Psi$ on the set of possible worlds $\Omega = \Omega(\Sigma)$.

- For subsets $\Theta$ of $\Sigma$, let $\mathcal{L}(\Theta)$ denote the propositional language defined by $\Theta$, with associated set of interpretations $\Omega(\Theta)$.

- Each interpretation $\omega \in \Omega$ can be written uniquely in the form $\omega = \omega^\Theta \omega^{\overline{\Theta}}$ with concatenated $\omega^\Theta \in \Omega(\Theta)$ and $\omega^{\overline{\Theta}} \in \Omega(\overline{\Theta})$, where $\overline{\Theta} = \Sigma \setminus \Theta$ is the complement of $\Theta$ in $\Sigma$.

- We also call $\omega^\Theta$ the restriction of $\omega$ to $\Theta$. 
Definition 2 (Marginalization of $\Psi$, $\Psi|_{\Theta}$)

Let $\Psi = (\Omega = \Omega(\Sigma), \preceq_\Psi)$ be an epistemic state given by a total preorder $\preceq_\Psi$ on $\Omega = \Omega(\Sigma)$, let $\Theta \subseteq \Sigma$. The marginalization of $\Psi$ on $\Theta$, denoted by $\Psi|_{\Theta}$, is defined via the induced total preorder on $\Omega(\Theta)$:

$$\Psi|_{\Theta} = (\Omega(\Theta), \preceq_{\Psi|_{\Theta}}), \ \omega_1^{\Theta} \preceq_{\Psi|_{\Theta}} \omega_2^{\Theta} \text{ iff } \omega_1^{\Theta} \preceq_\Psi \omega_2^{\Theta}.$$ 

Note that on the right hand side of the iff condition above $\omega_1^{\Theta}, \omega_2^{\Theta}$ are considered as propositions in the superlanguage $\mathcal{L}(\Omega)$, hence $\omega_1^{\Theta} \preceq_\Psi \omega_2^{\Theta}$ is well defined and allows for a more concise notation of $A$-filtering (with $\Theta = \Sigma(A)$):

$$\omega_1 \preceq_A \Psi \omega_2 \text{ iff } \omega_1^{\Theta} \preceq_{\Psi|_{\Theta}} \omega_2^{\Theta}.$$
Epistemic versions of (Q1) and (Q2)

(Q1) and (Q2) are equivalent to

(EQ1) Let $\omega_1, \omega_2 \in \Omega$ be possible worlds. If for all $i, 1 \leq i \leq n$, $\omega_1^{\Sigma_i} \in \text{min}(\Psi|_{\Sigma_i})$ or $\omega_1^{\Sigma_i} = \omega_2^{\Sigma_i}$, and there is $i, 1 \leq i \leq n$, such that $\omega_1^{\Sigma_i} \in \text{min}(\Psi|_{\Sigma_i})$ and $\omega_2^{\Sigma_i} \not\in \text{min}(\Psi|_{\Sigma_i})$, then $\omega_1 \prec_{\Psi} \omega_2$.

(EQ2) Let $\omega_1, \omega_2 \in \Omega$ be possible worlds. If for all $i, 1 \leq i \leq n$, both $\omega_1^{\Sigma_i}, \omega_2^{\Sigma_i} \in \text{min}(\Psi|_{\Sigma_i})$ or $\omega_1^{\Sigma_i} = \omega_2^{\Sigma_i}$, then $\omega_1 \approx_{\Psi} \omega_2$. 
\[ \text{Diff}(K, \omega) = \bigcup \{ F_i \in \mathcal{F} \mid \text{for some } \phi \in \mathcal{L}(F_i), K \models \phi \text{ and } \omega \models \neg \phi \} \]

\[ (K = \text{Cn}(\varphi_1, \ldots, \varphi_n), \mathcal{F} \text{ finest } K\text{-splitting}) \]

**Lemma 3**

\[ F_i \text{ occurs in } \text{Diff}(K, \omega) \text{ iff } \omega^i \not\models \varphi_i. \]

**Proposition 1**

\[ \text{Diff}(K, \omega) = \bigcup \{ F_i \in \mathcal{F} \mid \omega^i \not\models \varphi_i \}. \]
Epistemic versions of (R1) and (R2)

Theorem 4

Let $\Psi$ be an epistemic state defined on $\Omega(\Sigma)$ such that $\text{Bel}(\Psi) = \text{Cn}(A, B)$ with $\Sigma(A) \cap \Sigma(B) = \emptyset$. Let $C \in \mathcal{L}(\Sigma(A))$, let $\Psi * C$ be a revision of $\Psi$ by $C$. Let the conditions (ER1) and (ER2) be defined as follows:

\begin{align*}
(\text{ER1}) \quad & (\Psi * C)|_{\Sigma(A)} = \Psi|_{\Sigma(A)}.
(\text{ER2}) \quad & (\Psi * C)|_{\Sigma(A)} = (\Psi|_{\Sigma(A)}) * C.
\end{align*}

Then (ER1) and (ER2) imply (R1) and (R2) for $K = \text{Bel}(\Psi)$, i.e., (ER1) and (ER2) are the epistemic versions of (R1), (R2).
Syntax Splitting for epistemic states

Definition 5

Let \((\Sigma_1, \ldots, \Sigma_n)\) be a partition of \(\Sigma\), let \(\Psi\) be an epistemic state defined on \(\Sigma\). For each \(\omega \in \Omega(\Sigma)\), define that part of \(\omega\) that excludes exactly the \(i\)-part \(\omega^i\) as

\[
\hat{\omega}^i = \land_{j \neq i} \omega^j,
\]

where, as before, \(\omega^j = \omega^{\Sigma_j}\).

\(\Psi\) splits over \((\Sigma_1, \ldots, \Sigma_n)\) if the following condition holds for all \(i\):

\[
\omega_1 \preceq_{\Psi} \omega_2 \iff \omega_1^i \preceq_{\Psi|\Sigma_i} \omega_2^i \text{ whenever } \hat{\omega}_1^i = \hat{\omega}_2^i. \tag{1}
\]

Syntax splitting for epistemic states means that ceteris paribus (i.e., if \(\hat{\omega}_1^i = \hat{\omega}_2^i\) holds), relationships between marginalized worlds can be lifted.
Syntax Splitting for epistemic states – example

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>$\Psi$</th>
<th>$\Psi \ast A$</th>
<th>$\omega^1 \mid_{\Sigma_1}$</th>
<th>$\Psi \ast A \mid_{\Sigma_1}$</th>
<th>$\omega^2 \mid_{\Sigma_2}$</th>
<th>$\Psi \ast A \mid_{\Sigma_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$abc$</td>
<td>3</td>
<td>2</td>
<td>$ab$</td>
<td>1</td>
<td>0</td>
<td>$c$</td>
</tr>
<tr>
<td>$ab\bar{c}$</td>
<td>1</td>
<td>0</td>
<td>$ab$</td>
<td>1</td>
<td>0</td>
<td>$\bar{c}$</td>
</tr>
<tr>
<td>$\bar{a}bc$</td>
<td>4</td>
<td>3</td>
<td>$\bar{a}b$</td>
<td>3</td>
<td>2</td>
<td>$c$</td>
</tr>
<tr>
<td>$a\bar{b}c$</td>
<td>3</td>
<td>2</td>
<td>$a\bar{b}$</td>
<td>3</td>
<td>2</td>
<td>$\bar{c}$</td>
</tr>
<tr>
<td>$\bar{a}bc$</td>
<td>4</td>
<td>3</td>
<td>$\bar{a}b$</td>
<td>3</td>
<td>2</td>
<td>$c$</td>
</tr>
<tr>
<td>$\bar{a}b\bar{c}$</td>
<td>3</td>
<td>2</td>
<td>$\bar{a}b$</td>
<td>3</td>
<td>2</td>
<td>$\bar{c}$</td>
</tr>
<tr>
<td>$\bar{a}b\bar{c}$</td>
<td>2</td>
<td>3</td>
<td>$\bar{a}b$</td>
<td>0</td>
<td>1</td>
<td>$c$</td>
</tr>
<tr>
<td>$\bar{a}\bar{b}\bar{c}$</td>
<td>0</td>
<td>1</td>
<td>$\bar{a}b$</td>
<td>0</td>
<td>1</td>
<td>$\bar{c}$</td>
</tr>
</tbody>
</table>

$\Sigma = \{a, b, c\}$, $\Sigma_1 = \{a, b\}$, $\Sigma_2 = \{c\}$, $A = a \lor b$. $\Psi$ splits over $(\Sigma_1, \Sigma_2)$, but $\Psi \ast A$ does not split over $(\Sigma_1, \Sigma_2)$.
...and its relation to propositional syntax splitting

If the epistemic state $\Psi$ splits over $(\Sigma_1, \ldots, \Sigma_n)$, then

- $\text{Bel}(\Psi)$ splits over $(\Sigma_1, \ldots, \Sigma_n)$
- two of the three semantic conditions (Q1), (Q2) of [Peppas et al., 2015] are fulfilled.
A gap on the level of epistemic states

Let $\Psi$ be an epistemic state that splits over $(\Sigma_1, \ldots, \Sigma_n)$ of $\Sigma$, and let $C = \{C_1, \ldots, C_n\}$ with $C_i \in \mathcal{L}(\Sigma_i)$.

Lifting syntactical axioms (R1), (R2):

Marginalized Revision (MR) $\Psi \ast C|_{\Sigma_i} = (\Psi|_{\Sigma_i}) \ast C_i$.

Lifting semantic axioms (Q1)-(Q2):

Strong iterated P ($P^{it}$) $\Psi \ast C$ splits over $(\Sigma_1, \ldots, \Sigma_n)$.

$(\text{MR}) \iff (P^{it})$
Syntax splitting for OCFs

Definition 6

Let \((\Sigma_1, \ldots, \Sigma_n)\) be a partition of \(\Sigma\), let \(\kappa\) be an OCF defined on \(\Sigma\). \(\kappa\) ocf-splits over \((\Sigma_1, \ldots, \Sigma_n)\) iff there are OCFs \(\kappa_i\) defined on \(\Sigma_i\) such that

\[
\kappa(\omega) = \kappa(\omega^1 \ldots \omega^n) = \kappa_1(\omega^1) + \ldots + \kappa_n(\omega^n). \tag{2}
\]

In this case, we write \(\kappa = \kappa_1 \oplus \ldots \oplus \kappa_n\).

If \(\kappa = \kappa_1 \oplus \ldots \oplus \kappa_n\), then \(\kappa_i\) is the marginal of \(\kappa\) on \(\Sigma_i\), i.e., \(\kappa_i = \kappa|_{\Sigma_i}\).
Let \((\Sigma_1, \ldots, \Sigma_n)\) be a partition of \(\Sigma\), let \(\kappa\) be an OCF defined on \(\Sigma\) such that \(\kappa = \kappa_1 \oplus \ldots \oplus \kappa_n\) with marginal OCFs \(\kappa_i\) defined on \(\Sigma_i\). Let 
\[C = \{C_1, \ldots, C_n\}\] with \(C_i \in \mathcal{L}(\Sigma_i)\).

\[(P_{ocf})\] \(\kappa * C = (\kappa_1 * C_1) \oplus \ldots \oplus (\kappa_n * C_n)\).

\[(MR_{ocf})\] \(\kappa * C|_{\Sigma_i} = (\kappa|_{\Sigma_i})*C_i = \kappa_i * C_i\).

\[(P_{ocf})\text{ implies } (MR_{ocf})\]
\( (P_{ocf}) \) corresponds to System Independence of [Shore & Johnson, 1980] for the principle of minimum cross-entropy!
Propositional c-revisions

(cf. [GKI & Huvermann, 2017])

Definition 7 (Propositional c-revisions for OCFs)

Let $\kappa$ be an OCF specifying a prior epistemic state, and let $\mathcal{C} = \{C_1, \ldots, C_n\}$ represent new information. Then a (propositional) c-revision of $\kappa$ by $\mathcal{C}$ is given by the OCF

$$
\kappa \ast \mathcal{C}(\omega) = \kappa^*(\omega) = -\kappa(C_1 \ldots C_n) + \kappa(\omega) + \sum_{i=1, \omega \models C_i}^{n} \eta_i
$$

(3)

with non-negative integers $\eta_i$ satisfying

$$
\eta_i > \kappa(C_1 \ldots C_n) - \min_{\omega \models C_i} \{\kappa(\omega) + \sum_{j \neq i, \omega \models C_j} \eta_j\}.
$$

(4)

In this case, we write $\kappa^* = \kappa \ast \vec{\eta} \mathcal{R}$ with $\vec{\eta} = (\eta_1, \ldots, \eta_n)$. 
Theorem 8

Let \((\Sigma_1, \ldots, \Sigma_n)\) be a partition of \(\Sigma\), let \(C = \{C_1, \ldots, C_n\}\) with \(C_i \in L(\Sigma_i)\) be a set of propositions of \(L\). Let \(\kappa\) be an OCF defined on \(\Sigma\) such that \(\kappa = \kappa_1 \oplus \ldots \oplus \kappa_n\) with marginal OCFs \(\kappa_i\) defined on \(\Sigma_i\). Then any \(c\)-revision of \(\kappa\) by \(C\) satisfies \((P^{ocf})\). More precisely,

\[
\kappa \ast (\eta_1, \ldots, \eta_n) C = (\kappa_1 \ast \eta_1 C_1) \oplus \ldots \oplus (\kappa_n \ast \eta_n C_n),
\]

where the \(\eta_i\)'s are the impact factors associated with the propositions \(C_i\).
Example

Let $\Sigma = \{a, b, c, d\}$ with splitting $(\Sigma_1 = \{a\}, \Sigma_2 = \{b, c\}, \Sigma_3 = \{d\})$. $\kappa$ (as given in the table) ocf-splits over $(\Sigma_1, \Sigma_2, \Sigma_3)$ (the marginals $\kappa_i = \kappa|_{\Sigma_i}$, $i = 1, 2, 3$, can be found below). $\kappa$ is c-revised by the new information $C = \{\overline{a}, bc, d\}$ which is also split over $(\Sigma_1, \Sigma_2, \Sigma_3)$.

A multiple c-revision is given by

$$\kappa \ast C(\omega) = -6 + \kappa(\omega) + \begin{cases} 
0 & , \omega | = \overline{abcd} \\
\eta_1 & , \omega | = abcd \\
\eta_2 & , \omega | = \overline{ab}cd \\
\eta_3 & , \omega | = \overline{abc}d \\
\eta_1 + \eta_2 & , \omega | = \overline{abcd} \\
\eta_1 + \eta_3 & , \omega | = ab\overline{cd} \\
\eta_2 + \eta_3 & , \omega | = \overline{abc}d \\
\eta_1 + \eta_2 + \eta_3 & , \omega | = \overline{abc}d 
\end{cases}$$

The (Pareto) minimal parameters are $\eta_1 = 3, \eta_2 = 4, \eta_3 = 2$. 
### Example

<table>
<thead>
<tr>
<th>( \omega )</th>
<th>( \kappa )</th>
<th>( \kappa \ast C )</th>
<th>((\kappa \ast C)_{\text{min}})</th>
<th>((\kappa \ast \overline{abcd})_{\text{min}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{abcd} )</td>
<td>4</td>
<td>(-6 + 4 + \eta_1)</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>( \text{abcd} )</td>
<td>3</td>
<td>(-6 + 3 + \eta_1 + \eta_3)</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>( \text{ab\overline{cd}} )</td>
<td>2</td>
<td>(-6 + 2 + \eta_1 + \eta_2)</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>( \text{ab\overline{cd}} )</td>
<td>1</td>
<td>(-6 + 1 + \eta_1 + \eta_2 + \eta_3)</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>( \overline{abcd} )</td>
<td>1</td>
<td>(-6 + 1 + \eta_1 + \eta_2)</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( \overline{abcd} )</td>
<td>0</td>
<td>(-6 + 0 + \eta_1 + \eta_2 + \eta_3)</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>( \overline{a\overline{bcd}} )</td>
<td>3</td>
<td>(-6 + 3 + \eta_1 + \eta_2)</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>( \overline{a\overline{bcd}} )</td>
<td>2</td>
<td>(-6 + 2 + \eta_1 + \eta_2 + \eta_3)</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>( \overline{\overline{abcd}} )</td>
<td>6</td>
<td>(-6 + 6)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \overline{a\overline{bcd}} )</td>
<td>5</td>
<td>(-6 + 5 + \eta_3)</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>( \overline{a\overline{bcd}} )</td>
<td>4</td>
<td>(-6 + 4 + \eta_2)</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>( \overline{a\overline{bcd}} )</td>
<td>3</td>
<td>(-6 + 3 + \eta_2 + \eta_3)</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>( \overline{\overline{abcd}} )</td>
<td>3</td>
<td>(-6 + 3 + \eta_2)</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>( \overline{\overline{abcd}} )</td>
<td>2</td>
<td>(-6 + 2 + \eta_2 + \eta_3)</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>( \overline{\overline{abcd}} )</td>
<td>5</td>
<td>(-6 + 5 + \eta_2)</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>( \overline{\overline{abcd}} )</td>
<td>4</td>
<td>(-6 + 4 + \eta_2 + \eta_3)</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
Example (cont’d)

<table>
<thead>
<tr>
<th>$\Sigma_1$</th>
<th>$\kappa_1$</th>
<th>$\kappa_1 \ast \overline{a}$</th>
<th>$\Sigma_2$</th>
<th>$\kappa_2$</th>
<th>$\kappa_2 \ast bc$</th>
<th>$\Sigma_3$</th>
<th>$\kappa_3$</th>
<th>$\kappa_3 \ast d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0</td>
<td>1</td>
<td>$bc$</td>
<td>3</td>
<td>0</td>
<td>$d$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\overline{a}$</td>
<td>2</td>
<td>0</td>
<td>$\overline{b}c$</td>
<td>1</td>
<td>2</td>
<td>$\overline{d}$</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$\overline{b}c$</td>
<td>0</td>
<td>1</td>
<td>$\overline{b}c$</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>